

Properties and Attributes of Functions

9A Functions and Their Graphs

- 9-1 Multiple Representations of Functions
- 9-2 Piecewise Functions
- Lab Graph Piecewise Functions
- 9-3 Transforming Functions

MULTI-STEP TEST PREP

9B Functional Relationships

- 9-4 Operations with Functions
- 9-5 Functions and Their Inverses
- Lab Explore Differences and Ratios
- 9-6 Modeling Real-World Data

MULTI-STEP TEST PREP

COSMIC DEBRIS

Space missions have left more than 28,000 pieces of debris floating in space. You can analyze the debris trends by using functions and graphs.



Chapter Project Online

KEYWORD: MB7 ChProj

ARE YOU READY?

✓ Vocabulary

Match each term on the left with a definition on the right.

- | | |
|----------------|--|
| 1. translation | A. the statistical study of the relationship between variables |
| 2. slope | B. the constant rate of change of a linear function |
| 3. regression | C. the ratio between two sets of measurements |
| 4. correlation | D. a transformation that moves each point in a figure or graph the same distance in the same direction |
| | E. a measure of the strength and direction of the linear relationship between two variables |

✓ Connect Words and Algebra

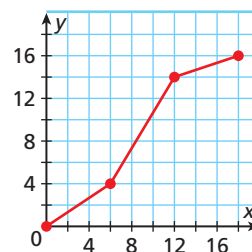
Write an equation to represent each situation.

- The cost of renting a recording studio is \$30 for the first hour and \$20 for each additional hour.
- The volume of water in a tank is equal to 30 gallons plus 8 gallons for every minute the pump is on.

✓ Line Graphs

Find each value for the graph of $f(x)$ shown.

- $f(6)$
- $f(15)$
- x such that $f(x) = 2$
- x such that $f(x) = 9$
- Find the slope of the line segment between $x = 6$ and $x = 12$.
- Find the slope of the line segment between $x = 12$ and $x = 18$.



✓ Multiply Binomials

Multiply. Then simplify.

- | | |
|----------------------------|-------------------------|
| 13. $(x - 6)(x + 4)$ | 14. $(6 - x)(4 - x)$ |
| 15. $(5x + 8)(2x - 7)$ | 16. $(x^2 - 7)(4x + 5)$ |
| 17. $(3x^2 + 8)(7x^2 + 8)$ | 18. $(x - 8)(x + 8)$ |

✓ Simplify Polynomial Expressions

Simplify.

- | | |
|---------------------------------|----------------------------------|
| 19. $8(3x^5) - (2x)^3(5x^2)$ | 20. $5(x + 3)^2 - 6(x + 3)$ |
| 21. $3x(4 - x^3) - 6x^2(x + 4)$ | 22. $3x^3(x^2 + 4) - x(x^4 - 5)$ |

Study Guide: Preview

Where You've Been

Previously, you

- studied different functions, graphs, and equations.
- transformed linear, quadratic, exponential, and radical functions.
- performed operations on many types of expressions.
- used linear, quadratic, and exponential functions to model real-world data.

In This Chapter

You will study

- multiple representations of functions.
- transforming piecewise functions.
- performing operations on functions and function inverses.
- using various functions to model real-world data.

Where You're Going

You can use the skills in this chapter

- in all of your future math classes, including Calculus and Statistics.
- in other classes, such as Health, Chemistry, Physics, and Economics.
- outside of school to model data and make predictions in sports, travel, and finance.

Key Vocabulary/Vocabulario

composition of functions	composición de funciones
one-to-one function	función uno a uno
piecewise function	función a trozos
step function	función escalón

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. One definition of the word *composition* is “the act or process of putting together.” How can you use this definition of *composition* to understand **composition of functions** in mathematics?
2. Imagine looking at a set of stairs from the side. Would a graph that looked like stairs represent a function? What might a **step function** look like?
3. Recall the definition of a function. What do you think a **one-to-one function** is? Give examples of functions from mathematics and from real life that are one-to-one functions and that are not one-to-one functions.

Reading Strategy: Read Problems for Understanding

Read a problem once to become aware of the concept being reviewed. Then read it again slowly and carefully to identify what the problem is asking. As you read, highlight key information given in the problem statement. When dealing with a multi-step problem, break the problem into parts and then make a plan to solve it.

- 19. Space Exploration** On Earth, the function $f(x) = \frac{6}{5}\sqrt{x}$ approximates the distance in miles to the horizon observed by a person whose eye level is x feet above the ground. The graph of the corresponding function for Mars is a horizontal stretch of f by a factor of about $\frac{9}{5}$. Write the corresponding function g for Mars, and use it to estimate the distance to the horizon for an astronaut whose eyes are 6 ft above Mars's surface.

Step	Question	Answer
Step 1	What concept is being reviewed?	transforming a rational function by changing its parameters
Step 2	What are you being asked to do?	- Rewrite the function to include the new parameter. - Evaluate the revised function for a given value.
Step 3	What is the key information needed to solve the problem?	- The function $f(x) = \frac{6}{5}\sqrt{x}$ represents the distance on Earth - The function for Mars is a horizontal stretch by a factor of $\frac{9}{5}$ - The astronaut's eye level on Mars is 6 ft.
Step 4	What is my plan to solve this multi-part problem?	- Revise the given function to account for horizontal stretch on Mars. - Evaluate the revised function for $x = 6$.

Try This

For each problem, complete each step in the four-step method described above.

- A rectangle has a length of $(x + 5)$ units and a width of $(x + 4)$ units. Write and graph a rational function R to represent the ratio of the area to the perimeter. Identify a reasonable domain and range of the function.
- The diameter d (in inches) of a rope needed to lift w tons is given by $d = \frac{\sqrt{15w}}{\pi}$. How much more can be lifted with a rope 1.25 inches in diameter than with a rope 0.75 inch in diameter?



9-1

Multiple Representations of Functions

Objectives

Translate between the various representations of functions.

Solve problems by using the various representations of functions.

Who uses this?

An amusement park manager can use representations of functions, such as graphs and tables, to analyze ticket sales. (See Example 1.)

An amusement park manager estimates daily profits by multiplying the number of tickets sold by 20. This verbal description is useful, but other representations of the function may be more useful.



Equation

$$p = 20n$$

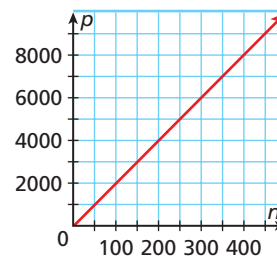
or

$$p(n) = 20n$$

Table

n	p
50	1000
100	2000
150	3000
200	4000

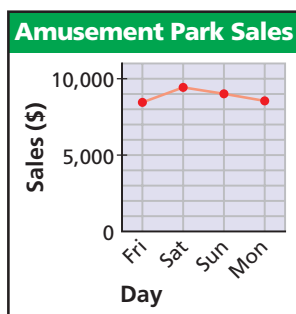
Graph



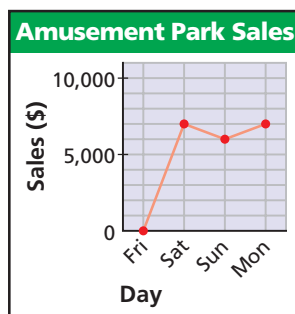
These different representations can help the manager set, compare, and predict prices.

EXAMPLE 1 Business Application

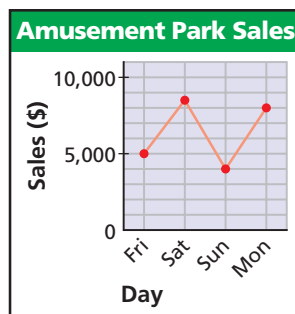
A manager at an amusement park monitors the ticket sales at the park over a four-day weekend. Match each situation to one of the following graphs. Sketch a possible graph of the situation if the situation does not match any of the given graphs.



Graph 1



Graph 2



Graph 3

- A** The park was closed on Friday for repairs.
graph 2 *The graph shows no ticket sales on Friday.*
- B** The park hosted a big concert on Saturday and a parade on Monday.
graph 3 *The graph shows increased ticket sales on Saturday and Monday.*
- C** The park was very busy during the holiday weekend.
graph 1 *The graph shows high ticket sales every day.*



What if...? Sketch a possible graph to represent the following.

- 1a. The weather was beautiful on Friday and Saturday, but it rained all day on Sunday and Monday.
- 1b. Only $\frac{1}{2}$ of the rides were running on Friday and Sunday.

Because each representation of a function (words, equation, table, or graph) describes the same relationship, you can often use any representation to generate the others.

EXAMPLE 2 Recreation Application

Kurt is rappelling down a 500-foot cliff at a rate of 6 feet per second. Create a table, equation, and graph to represent Kurt's height from the ground with relation to time. When will Kurt reach the ground?

Step 1 Create a table.

Let t be the time in seconds and h be Kurt's height, in feet, from the ground.

Kurt begins at a height of 500 feet, and the height decreases by 6 feet each second.

t	h
0	500
1	494
2	488
3	482
4	476
5	470

$500 - 6$
 $500 - 6(2)$
 $500 - 6(3)$
 $500 - 6(4)$
 $500 - 6(5)$

Step 2 Write an equation.

Height	is equal to	500	minus	6 feet per second.
h	=	500	−	$6t$

Step 3 Find the intercepts and graph the equation.

h -intercept: 500

Solve for t when $h = 0$.

$$h = 500 - 6t$$

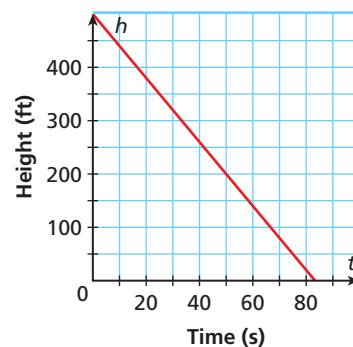
$$0 = 500 - 6t$$

$$-500 = -6t$$

$$t = \frac{-500}{-6} = 83\frac{1}{3}$$

t -intercept: $83\frac{1}{3}$

Kurt will reach the ground after $83\frac{1}{3}$ seconds.



2. The table shows the height, in feet, of an arrow in relation to its horizontal distance from the archer. Create a graph, an equation, and a verbal description to represent the height of the arrow with relation to its horizontal distance from the archer.

Arrow Distance and Height						
Distance from Archer (ft)	0	75	150	225	300	375
Height (ft)	6.55	59.80	90.55	98.80	84.50	47.50



Translating Between Multiple Representations	
When given a(n)...	Try to...
Table	- Find finite differences or ratios to determine which parent function best describes the data. - Graph points as ordered pairs and look for a pattern. - Match the data to the related parent function, if applicable, and perform a regression.
Graph	- Identify which parent function the graph most resembles, and then use key points (intercepts, maxima, minima, and so on) from the graph to help write an equation. - Locate several points on the graph and write them in a table. - Use slope; increasing, decreasing, or constant intervals; and intercepts to write a verbal description.
Equation	- Make a table of values. You may use a graphing calculator. - Make a graph by using transformations of parent functions or a graphing calculator.
Verbal Description	- Identify dependent and independent variables, and write an algebraic equation. - Generate a table of values by using the pattern described. - Sketch a graph of the situation by using hints from the description about increasing, decreasing, or constant intervals, as well as intercepts.

EXAMPLE 3 Using Multiple Representations to Solve Problems

- A** Stacy runs three days a week at a track. Stacy starts keeping time when she starts warming up and notes after every 2 laps how long she has been at the track. The table shows the times for several laps. Use a graph and an equation to find the time it will take Stacy to run 20 laps.

Stacy's Time	
Laps	Time (min)
2	13
4	16
6	19
8	22
10	25

Step 1 Graph the data.

The data appear to be linear.

Step 2 Write a linear equation.

Let x = the number of laps and
 y = the time in minutes.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16 - 13}{4 - 2} = \frac{3}{2} \quad \text{Find the slope. Use any two points.}$$

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

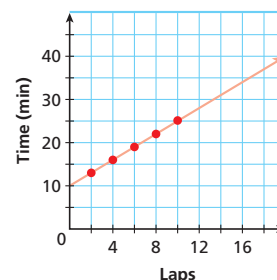
$$y - 13 = \frac{3}{2}(x - 2) \quad \text{Use (2, 13) and slope } \frac{3}{2}.$$

$$y = \frac{3}{2}x + 10 \quad \text{Simplify.}$$

Step 3 Evaluate the function for 20 laps.

$$y = \frac{3}{2}(20) + 10 = 40.$$

It will take Stacy 40 minutes to complete 20 laps.



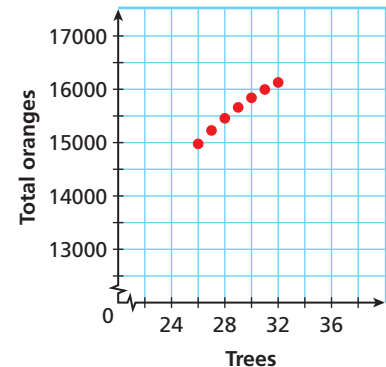
Remember!

The point-slope form of the equation of a line is
 $y - y_1 = m(x - x_1)$,
 where m is the slope and (x_1, y_1) is a point on the line.
 (Lesson 2-4)

- B** The owner of an orange grove finds that if 26 trees are planted per acre, each mature tree yields about 576 oranges per year. For each additional tree planted per acre, the number of oranges produced annually by each tree decreases by 12. Use a table, a graph, and an equation to find how many trees per acre should be planted to maximize the yield per acre.

Make a table for an acre of orange trees. Because the orchard owner is interested in the total number of oranges, make a graph by using trees t as the independent variable and total oranges as the dependent variable.

Orange Tree Yield		
Trees	Oranges per Tree	Total Oranges
26	576	14,976
27	564	15,228
28	552	15,456
29	540	15,660
30	528	15,840
31	516	15,996
32	504	16,128



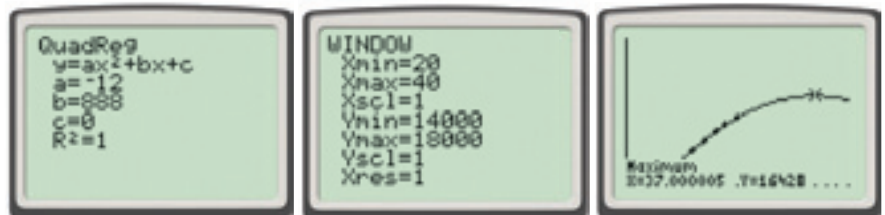
The data do not appear to be linear, so check finite differences.

Total oranges 14,976 15,228 15,456 15,660 15,840 15,996 16,128

First differences 252 228 204 180 156 132

Second differences -24 -24 -24 -24 -24

Because the second differences are constant, a quadratic model is appropriate. Use a graphing calculator to perform a quadratic regression on the data.



The equation $y = -12x^2 + 888x$ models the data, and the graph appears to fit. Use the **TRACE** or **MAXIMUM** feature to identify the maximum orange yield.

The maximum occurs when 37 trees are planted on each acre.

Remember!

First differences are constant in linear functions. Second differences are constant in quadratic functions. (Lesson 5-9)

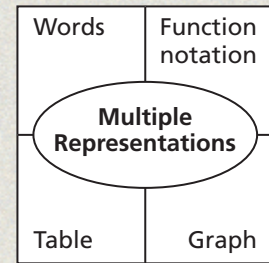


3. Bartolo opened a new sporting goods business and has recorded his sales each week. To break even, Bartolo needs to sell \$48,000 worth of merchandise in a week. Assuming the sales trend continues, use a graph and an equation to find the number of weeks before Bartolo breaks even.

Bartolo's Sales	
Week	Sales (\$)
1	25,000
2	27,500
3	30,250
4	33,275
5	36,603

THINK AND DISCUSS

1. Explain how to use a table to help create an equation for a set of data.
2. Give an example of a real-world situation where a graph might be the most useful representation of a set of data.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box give an example.



9-1

Exercises



Homework Help Online

KEYWORD: MB7 9-1

Parent Resources Online

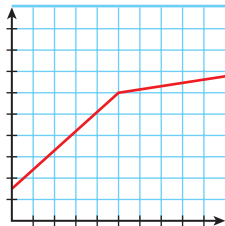
KEYWORD: MB7 Parent

GUIDED PRACTICE

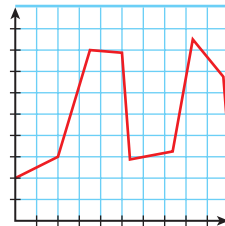
SEE EXAMPLE 1

p. 654

Match each situation to its corresponding graph. Sketch a possible graph of the situation if the situation does not match any of the given graphs.



Graph A



Graph B



Graph C



Graph D

1. Due to a product recall, a company's profits drop sharply into a loss but rebound a few weeks later.
2. The value of a car declines as the car gets older.
3. A souvenir shop's sales are seasonal, with high sales in summer and winter and low sales in spring and fall.
4. An airplane ascends to a peak height of 30,000 feet and then descends to a cruising altitude of 24,000 feet.

SEE EXAMPLE 2

p. 655

5. **Education** Part-time students at a university must pay an enrollment fee of \$179.35, plus \$218.40 per credit hour. Create a table, an equation, and a graph that give the total cost of enrollment as a function of credit hours.

SEE EXAMPLE 3

p. 656

6. **Recreation** Claire is hiking up the South Kaibab Trail at the Grand Canyon. The table shows Claire's altitude above sea level every 15 minutes after she starts to hike. Use a graph and an equation to find how long it will take Claire to reach the rim of the canyon at 7260 feet.

Claire's Altitude	
Time (min)	Altitude (ft)
15	2940
30	3240
45	3540
60	3840
75	4140

PRACTICE AND PROBLEM SOLVING

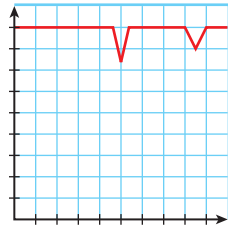
Independent Practice

For Exercises	See Example
7–10	1
11–12	2
13–14	3

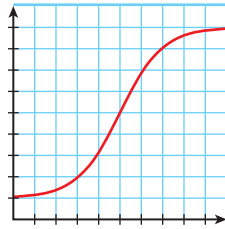
Extra Practice

Skills Practice p. S20
Application Practice p. S40

Match each situation to its corresponding graph. Sketch a possible graph of the situation if the situation does not match any of the given graphs.



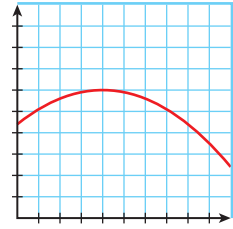
Graph A



Graph B



Graph C



Graph D

- The sales of lift tickets at a ski resort are highest at the beginning and end of the year.
- The attendance at a pop singer's concerts is steadily high except on two nights.
- The population of a city peaked in the 1980s and has been decreasing slowly but steadily in the years since.
- Sales of a new type of cell phone increase rapidly and then level off.

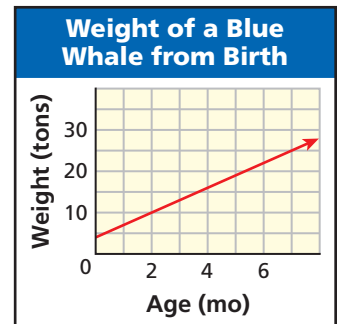
- Health** Carl has a severe fever, so his doctor advises him to take his temperature every 4 hours until it falls below 100°F . The table shows Carl's temperature with relation to time. Create a graph, an equation, and a verbal description to represent Carl's temperature with relation to time. When will Carl's temperature drop below 100°F ?

Carl's Temperature	
Time (h)	Temperature ($^{\circ}\text{F}$)
0	101.10
4	102.82
8	103.78
12	103.98
16	103.42
20	102.10

- Transportation** A truck begins a trip of 1675 miles. The truck averages 55 miles per hour, including stops. Create a table, a graph, and an equation to represent the distance that the truck has left to travel with relation to time.

- Whales** Researchers are studying the growth of a young blue whale. The graph shows the approximate weight of the whale from birth to 8 months.

- Find an equation for the weight of the whale as a function of time, and describe the relationship in words.
- Will the weight of the whale continue to increase by the same amount each month? Explain your answer.



- Business** Alex is painting a house. When Alex starts work on Monday morning, there are 2452 square feet of surface area that remain to be painted. Alex can paint 64 square feet of surface area in an hour.
 - Write an equation for the amount of surface area that Alex has left to paint after t hours.
 - If Alex works for 40 hours a week, will he be able to finish painting the house in a week?
- Sports** The owners of a minor league hockey team have found that when they charge \$12 for a lower-level seat, they average 800 fans per game. For every \$1 increase in ticket price, the attendance decreases by an average of 50 people. Find the ticket price that will maximize revenue for the team's owners.



Whales



Blue whales are the largest and loudest animals on Earth. Their calls can reach intensity levels of 180 decibels, louder than a jet airplane.



Geometry Each container is filled at a constant rate. Sketch a graph of the height of water in each container as a function of time.

16.



17.

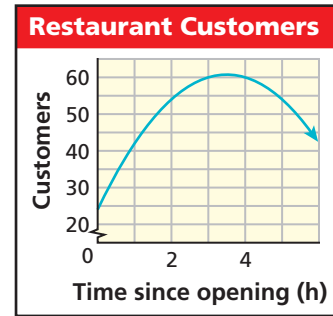


18.



19. **Business** In order to better manage her restaurant, Rita counts the number of people who are in the restaurant at the end of every hour after the restaurant is opened. The results are shown in the graph.

- Write a function for the graph.
- According to your function, what was the maximum number of customers in Rita's restaurant on this evening?
- When will there be no customers in Rita's restaurant?



20. **Hobbies** Susan collects antique dolls. In 2005, her collection contained 6 dolls. She plans to double the number of dolls in her collection every year. Use a table, a graph, and an equation to determine when Susan will have more than 100 dolls in her collection.

21. **Forestry** The *Sorbus aucuparia*, or mountain ash tree, typically grows to the heights shown in the table.
- Create a graph of height versus time.
 - Write a function that models the height.
 - During which year would you expect the height to reach 18 ft?

Growth of Mountain Ash	
Year	Height (ft)
1	4
3	7
6	10
10	13



22. **Write About It** Describe a different situation in which you would find each representation of a function, including a table, a graph, and an equation, useful.
23. **Critical Thinking** When would a graph give you more evidence about a relationship than a table would? When would a table give more evidence than a graph would?

MULTI-STEP TEST PREP

24. This problem will prepare you for the Multi-Step Test Prep on page 680.

A group of people stand in a circle and hold hands. One person squeezes the hand of the person on her left, who then squeezes the hand of the next person, and so on. The table shows the time that it takes the signal to go all the way around the circle.

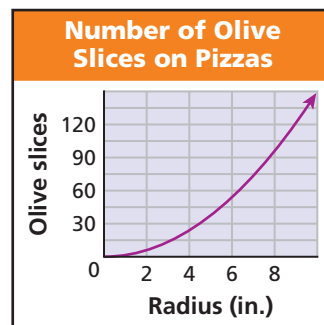
- Create a graph of time versus the number of participants.
- Write a function that models the situation.
- Suppose the signal takes about a minute to go around the circle. How many participants are there?

Time for Hand Squeeze to Complete a Cycle	
People	Time (s)
5	2.10
8	3.36
14	5.88
23	9.66



25. **Business** This graph shows data on the number of olive slices on pizzas of different radii. Let r represent the radius of the pizza and n represent the number of olive slices. Identify the equation that best represents the relationship between the radius and the number of olive slices.

(A) $n = -\frac{3}{2}r^2$ (C) $n = \frac{3}{2}r^2$
 (B) $n = -6r$ (D) $n = 6r$



26. A charity is selling American flags to celebrate Independence Day. The charity's profit in dollars is modeled by $p = \frac{1}{2}n$, where n is the number of flags sold. Which of the following choices identifies the same function?

(A) The profit is \$2 per flag.

(C) For every 2 flags sold, the profit is \$1.

(B)

n	1	2	3
p	2	4	6

(D)

n	1	2	3	4
p	0.5	1.5	2.5	3.5

27. **Short Response** Which type of function would best model the cost for carpeting a square room as a function of the room's width? Explain your answer.

CHALLENGE AND EXTEND

Write an equation and create a graph for each situation described.

28. The volume of a box for a glass decoration is found by doubling the radius of the decoration, raising it to the third power, and then adding 10.
29. The total cost of an item at a sale is found by taking off a 20% discount, subtracting a \$10-off coupon, and adding 6.5% sales tax.

30. **Finance** Sharmila was able to save \$500 from her summer job. She put the money into a mutual fund. This table shows how the value of Sharmila's money has grown.

- a. Write an appropriate model for the amount that Sharmila will have in this mutual fund after t years.
- b. Use your model to predict when Sharmila will have \$2000 in the mutual fund.

Sharmila's Mutual Fund	
Year	Value (\$)
1	545.00
2	594.00
3	647.51
4	705.79
5	769.31

SPIRAL REVIEW

Find the vertex of each function. (Lesson 2-9)

31. $f(x) = |x + 3| - 4$

32. $f(x) = -|x - 1| - 2$

33. Tim paid \$200 to have his lawn fertilized. The lawn-care company charged \$0.25 per square foot. If Tim's lawn is a rectangle with a length that is twice its width, find the dimensions of the lawn. (Lesson 5-4)

Graph each function, and identify its domain and range. (Lesson 8-7)

34. $f(x) = \sqrt{x + 3}$

35. $f(x) = 3\sqrt{x - 1}$

9-2

Piecewise Functions

Objectives

Write and graph piecewise functions.

Use piecewise functions to describe real-world situations.

Vocabulary

piecewise function
step function

Why learn this?

You can use piecewise functions to model an athlete's performance in a triathlon. (See Example 4.)

A **piecewise function** is a function that is a combination of one or more functions. The rule for a piecewise function is different for different parts, or pieces, of the domain. For instance, movie ticket prices are often different for different age groups. So the function for movie ticket prices would assign a different value (ticket price) for each domain interval (age group).

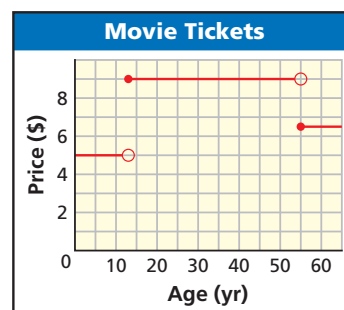


EXAMPLE 1 Entertainment Application

Create a table and a verbal description to represent the graph.

Step 1 Create a table.

Because the endpoints of each segment of the graph identify the intervals of the domain, use the endpoints and points close to them as the domain values in the table.



Movie Tickets	
Age	Price (\$)
0–12	5.00
13–54	9.00
55+	6.50

The domain of the function is divided into three intervals:

Ages 12 and under $\longrightarrow [0,13)$
 Ages 13 and under 55 $\longrightarrow [13,55)$
 Ages 55 and over $\longrightarrow [55,\infty)$

Step 2 Write a verbal description.

Use the domain intervals and the prices from the table.

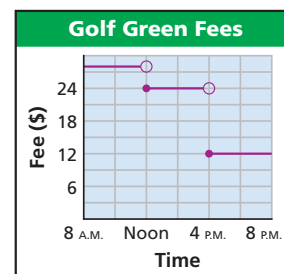
Movie tickets are \$5.00 for children ages 12 and under, \$9.00 for people ages 13 through 54, and \$6.50 for seniors ages 55 years and older.

Remember!

When using interval notation, square brackets $[]$ indicate an included endpoint, and parentheses $()$ indicate an excluded endpoint. (Lesson 1-1)



1. Create a table and a verbal description to represent the graph.



A piecewise function that is constant for each interval of its domain, such as the ticket price function, is called a **step function**. You can describe piecewise functions with a function rule. The rule for the movie ticket prices from Example 1 is shown.

$$f(x) = \begin{cases} 5 & \text{if } 0 < x < 13 \\ 9 & \text{if } 13 \leq x < 55 \\ 6.5 & \text{if } x \geq 55 \end{cases}$$

Read this as “*f* of *x* is 5 if *x* is greater than 0 and less than 13, 9 if *x* is greater than or equal to 13 and less than 55, and 6.5 if *x* is greater than or equal to 55.”

To evaluate any piecewise function for a specific input, find the interval of the domain that contains that input and then use the rule for that interval.

EXAMPLE 2 Evaluating a Piecewise Function

Evaluate each piecewise function for $x = -2$ and $x = 5$.

A $f(x) = \begin{cases} -5 & \text{if } x \leq 0 \\ 4 & \text{if } 0 < x \leq 3 \\ 12 & \text{if } x > 3 \end{cases}$

$$f(-2) = -5$$

Because $-2 \leq 0$, use the rule for $x \leq 0$.

$$f(5) = 12$$

Because $5 > 3$, use the rule for $x > 3$.

B $g(x) = \begin{cases} 3x + 4 & \text{if } x < 5 \\ x^2 - 3 & \text{if } x \geq 5 \end{cases}$

$$g(-2) = 3(-2) + 4 = -2$$

Because $-2 < 5$, use the rule for $x < 5$.

$$g(5) = 5^2 - 3 = 22$$

Because $5 \geq 5$, use the rule for $x \geq 5$.



Evaluate each piecewise function for $x = -1$ and $x = 3$.

2a. $f(x) = \begin{cases} 12 & \text{if } x < -3 \\ 15 & \text{if } -3 \leq x < 6 \\ 20 & \text{if } x \geq 6 \end{cases}$ **2b.** $g(x) = \begin{cases} 3x^2 + 1 & \text{if } x < 0 \\ 5x - 2 & \text{if } x \geq 0 \end{cases}$

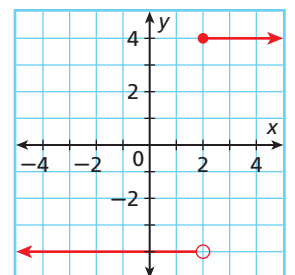
You can graph a piecewise function by graphing each piece of the function.

EXAMPLE 3 Graphing Piecewise Functions

Graph each function.

A $f(x) = \begin{cases} -4 & \text{if } x < 2 \\ 4 & \text{if } x \geq 2 \end{cases}$

The function is composed of two constant pieces that will be represented by horizontal rays. Because the domain is divided at $x = 2$, evaluate both branches of the function at $x = 2$. The function is -4 when $x < 2$, so plot the point $(2, -4)$ with an open circle and draw a horizontal ray to the left. The function is 4 when $x \geq 2$, so plot the point $(2, 4)$ with a solid dot and draw a horizontal ray to the right.



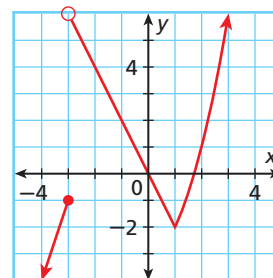
Graph each function.

B $g(x) = \begin{cases} 3x + 8 & \text{if } x \leq -3 \\ -2x & \text{if } -3 < x < 1 \\ x^2 - 3 & \text{if } x \geq 1 \end{cases}$

The function is composed of two linear pieces and a quadratic piece. The domain is divided at $x = -3$ and $x = 1$.

Use a table of values to graph each piece.

x	$g(x) = 3x + 8$	$g(x) = -2x$	$g(x) = x^2 - 3$
-4	-4		
-3	-1	6	
-2		4	
-1		2	
0		0	
1		-2	-2
2			1
3			6



Add a closed circle at $(-3, -1)$ and an open circle at $(-3, 6)$ so that the graph clearly shows the function value when $x = -3$.

No circle is required at $(1, -2)$ because the function is connected at that point.



Graph each function.

3a. $f(x) = \begin{cases} 4 & \text{if } x \leq -1 \\ -2 & \text{if } x > -1 \end{cases}$

3b. $g(x) = \begin{cases} -3x & \text{if } x < 2 \\ x + 3 & \text{if } x \geq 2 \end{cases}$

Notice that piecewise functions are not necessarily *continuous*, meaning that the graph of the function may have breaks or gaps.

To write the rule for a piecewise function, determine where the domain is divided and write a separate rule for each piece. Combine the pieces by using the correct notation.

Student to Student

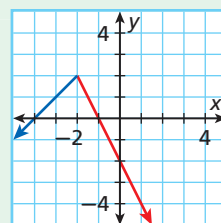
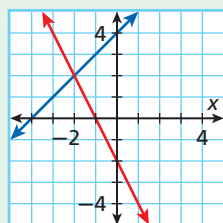
Graphing Piecewise Functions



Mateo Morales
Lee High School

When I graph a piecewise function, I like to graph each piece like it's a separate function. Then I go back and erase the parts that are outside of the restricted domain.

Example: $f(x) = \begin{cases} x + 4 & \text{if } x < -2 \\ -2x & \text{if } x \geq -2 \end{cases}$



EXAMPLE 4 Sports Application

Remember!

The distance formula $d = rt$ can be rearranged to find rates: $r = \frac{d}{t}$.

David is completing a 100-mile triathlon. He swims 2 miles in 1 hour, then bikes 80 miles in 4 hours, and finally he runs 18 miles in 3 hours. Sketch a graph of David's distance versus time. Then write a piecewise function for the graph.

Step 1 Make a table to organize the data. Use the distance formula to find David's rate for each leg of the race.

David's Race			
Activity	Time (h)	Distance (mi)	Rate (mi/h)
Swimming	1	2	2
Biking	4	80	20
Running	3	18	6

Step 2 Because time is the independent variable, determine the intervals for the function.

Swimming: $0 \leq t \leq 1$

Biking: $1 < t \leq 5$

Running: $5 < t \leq 8$

He swims for 1 hour.

He bikes for the next 4 hours.

He runs the final 3 hours.

Step 3 Graph the function.

After 1 hour, David has covered 2 miles. On the next leg, he reaches a distance of 82 total miles after 5 total hours. Finally, he completes the 100 miles after 8 hours.

Step 4 Write a linear function for each leg.

Use point-slope form:

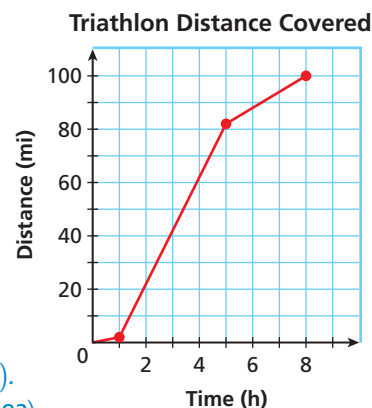
$$y - y_1 = m(x - x_1).$$

Swimming: $d = 2t$ Use $m = 2$ and $(0, 0)$.

Biking: $d = 20t - 18$ Use $m = 20$ and $(5, 82)$.

Running: $d = 6t + 52$ Use $m = 6$ and $(8, 100)$.

$$\text{The function rule is } d(t) = \begin{cases} 2t & \text{if } 0 \leq t \leq 1 \\ 20t - 18 & \text{if } 1 < t \leq 5 \\ 6t + 52 & \text{if } 5 < t \leq 8 \end{cases}$$



4. Shelly earns \$8 an hour. She earns \$12 an hour for each hour over 40 that she works. Sketch a graph of Shelly's earnings versus the number of hours that she works up to 60 hours. Then write a piecewise function for the graph.

THINK AND DISCUSS

- Tell whether it is possible to have a continuous step function.
- GET ORGANIZED** Copy and complete the graphic organizer. Describe the domain and range for each function. Then include an example.

Function	Domain	Range	Example
Piecewise			
Step			





GUIDED PRACTICE

1. **Vocabulary** How are step functions related to piecewise functions?

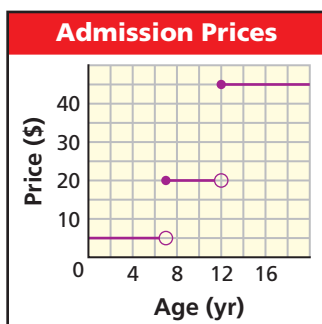
Create a table and a verbal description to represent each graph.

SEE EXAMPLE 1

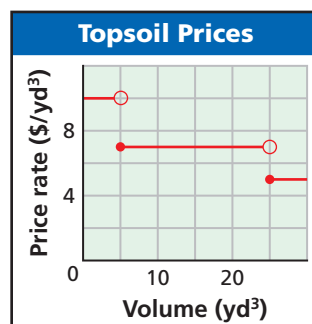
1

p. 662

2.



3.



SEE EXAMPLE 2

2

p. 663

Evaluate each piecewise function for $x = -6$ and $x = 3$.

$$4. f(x) = \begin{cases} -8 & \text{if } x \leq -5 \\ 0 & \text{if } -5 < x < 5 \\ 5 & \text{if } x \geq 5 \end{cases}$$

$$5. g(x) = \begin{cases} 5x - 9 & \text{if } x < 2 \\ 4 - x^2 & \text{if } x \geq 2 \end{cases}$$

SEE EXAMPLE 3

3

p. 663

Graph each function.

$$6. f(x) = \begin{cases} 7 & \text{if } x < -2 \\ -2 & \text{if } x \geq -2 \end{cases}$$

$$7. g(x) = \begin{cases} -2x + 8 & \text{if } x \leq 4 \\ \frac{1}{2}x & \text{if } x > 4 \end{cases}$$

SEE EXAMPLE 4

4

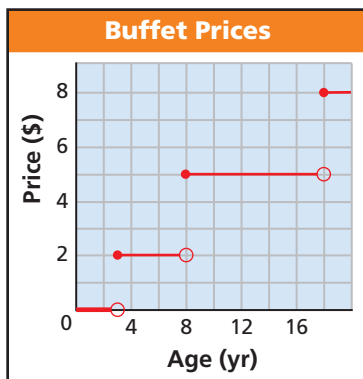
p. 665

8. The cost of renting a canoe is \$20 for the first 4 hours and \$3 per hour for additional hours. Sketch a graph of the cost of renting a canoe from 0 to 8 hours. Then write a piecewise function for the graph.

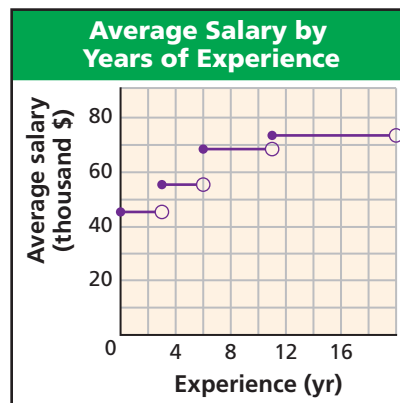
PRACTICE AND PROBLEM SOLVING

Create a table and a verbal description to represent each graph.

9.



10.



Evaluate each piecewise function for $x = -2$, $x = 2$, and $x = 6$.

$$11. g(x) = \begin{cases} 9x - 2 & \text{if } x < -3 \\ x^2 - 3 & \text{if } -3 \leq x < 1 \\ 5 & \text{if } x \geq 1 \end{cases}$$

$$12. f(x) = \begin{cases} 12 - 9x & \text{if } x \leq 0 \\ x^2 + 3x & \text{if } 0 < x < 3 \\ 4^x & \text{if } x \geq 3 \end{cases}$$

Independent Practice

For Exercises See Example

9–10 1

11–12 2

13–14 3

15 4

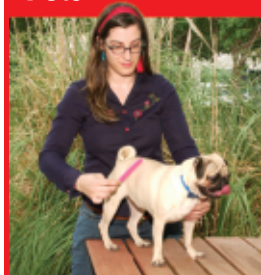
Extra Practice

Skills Practice p. S20

Application Practice p. S40



Pets



Dog grooming is a \$2 billion industry in the United States. There is evidence that there were dog-grooming parlors in France as early as the 1700s.

Graph each function.

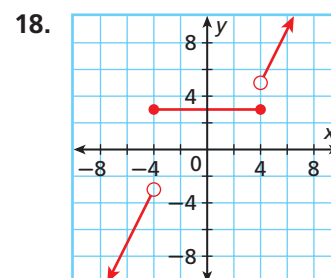
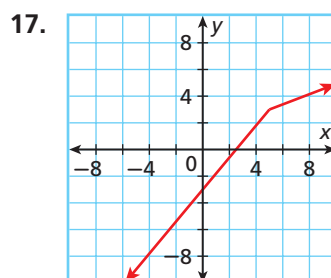
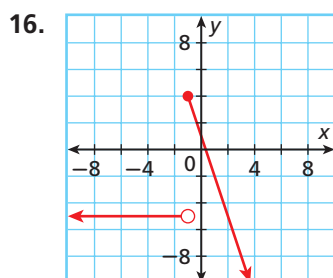
$$13. f(x) = \begin{cases} \frac{3}{4}x + 1 & \text{if } x < 4 \\ \frac{3}{4}x - 2 & \text{if } x \geq 4 \end{cases}$$

$$14. g(x) = \begin{cases} -2x - 5 & \text{if } x < -2 \\ x^2 - 3 & \text{if } x \geq -2 \end{cases}$$

15. **Pets** A dog groomer charges different prices based on the weight of the dog. Sketch a graph of the cost of grooming a dog from 0 to 100 pounds. Then write a piecewise function for the graph.

Grooming Prices	
Weight (lb)	Price (\$)
15 and under	30
Over 15 and up to 50	50
Over 50	75

Write a piecewise function for each graph.



19. **Parking** A parking garage charges \$6 for the first 4 hours that a car is parked in the lot. After that, the garage charges an additional \$3 an hour. Write a piecewise function for the cost of parking a car in this garage for x hours.

20. **Travel** Derek and his friends drove from San Francisco to Lake Tahoe to go skiing. The average speed that they traveled during each leg of the trip is shown on the map. They drove 30 min in the city, 3 h on the highway, and 30 min up the mountain.

- Write a piecewise function to represent the distance that Derek traveled during his 4 h trip.
- Graph the function.
- What if...?** How much longer would the trip have taken if Derek had averaged 50 mi/h on the highway?

Average Speed



Write each absolute-value function as a piecewise function.

$$21. f(x) = |x|$$

$$22. g(x) = |x - 4|$$

$$23. h(x) = 2|x| - 4$$

24. **Shipping** An overnight delivery service charges \$11 for a package that weighs 2 pounds or less. The delivery service charges \$3 for each additional pound. Sketch a graph of the cost of shipping a package from 0 to 8 pounds. Then write a piecewise function for the graph.

Graph each function.

$$25. h(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x \leq 0 \\ 2^x - 4 & \text{if } 0 < x \leq 3 \\ 2x - 2 & \text{if } x > 3 \end{cases}$$

$$26. h(x) = \begin{cases} -3 & \text{if } x \leq 0 \\ 3^x - 4 & \text{if } x > 0 \end{cases}$$



27. This problem will prepare you for the Multi-Step Test Prep on page 680.

A human chain is formed by 60 people standing with their arms outstretched, each holding the hand of the person on either side. The first 30 people in the chain have arm spans of 6 feet. The next 30 people have arm spans of 5.5 feet. At the word “go,” the first person squeezes the hand of the second person, then the second person squeezes the hand of the third, and so on. Assume that each person takes $\frac{1}{3}$ second to pass along the signal.

- Write a piecewise function for the distance that the signal travels in t seconds.
- Does the signal travel faster in the first half of the chain or the second half? How is this shown in the function?

Find the domain and range of each piecewise function.

28. $f(x) = \begin{cases} -\frac{5}{2}x - 2 & \text{if } x \leq -2 \\ -x - 5 & \text{if } x > -2 \end{cases}$

29. $g(x) = \begin{cases} x^2 - 2x - 3 & \text{if } x < 4 \\ 3x - 7 & \text{if } x \geq 4 \end{cases}$

30. **Sales** Mary works at a jewelry store. She receives a base salary every week plus a commission based on how much she sells. Mary’s income function can be modeled by

$$P(x) = \begin{cases} 400 + 0.06x & \text{if } 0 \leq x \leq 5000 \\ 700 + 0.09(x - 5000) & \text{if } x > 5000 \end{cases}, \text{ where } P(x) \text{ is her income and } x \text{ is}$$

the amount of her sales in dollars.

- Write a description of Mary’s income function.
 - How much will Mary earn in a week in which she sells \$4000 worth of jewelry?
 - Find the value of the jewelry that Mary must sell in a week if she wants to earn \$900 for that week.
31. **Critical Thinking** Why would a piecewise function best describe the height of an elevator t seconds after it leaves the bottom floor of a building? Would the piecewise function also be a step function?
32. **Write About It** Explain why piecewise functions are often good for representing real-world situations. Include at least two examples.



33. A car rental agency charges \$15 a day for driving a car 200 miles or less. If a car is driven over 200 miles, the renter must pay \$0.05 for each mile over 200 driven. Which of the following functions represents the cost to drive a car from this agency x miles in a day?

(A) $C(x) = \begin{cases} 15 & \text{if } 0 \leq x \leq 200 \\ 0.05x & \text{if } x > 200 \end{cases}$

(C) $C(x) = \begin{cases} 15 & \text{if } 0 \leq x \leq 200 \\ 15 + 0.05(x - 200) & \text{if } x > 200 \end{cases}$

(B) $C(x) = \begin{cases} 0.05 & \text{if } 0 \leq x \leq 200 \\ 15x & \text{if } x > 200 \end{cases}$

(D) $C(x) = \begin{cases} 15 & \text{if } 0 \leq x \leq 200 \\ 15 + 0.05x & \text{if } x > 200 \end{cases}$

34. Which of the following is a continuous function?

(F) $f(x) = \begin{cases} 3x - 4 & \text{if } x < 0 \\ -1 & \text{if } x \geq 0 \end{cases}$

(H) $h(x) = \begin{cases} x^2 & \text{if } x < -2 \\ 2x & \text{if } x \geq -2 \end{cases}$

(G) $g(x) = \begin{cases} 5x - 4 & \text{if } x < 3 \\ 2x + 5 & \text{if } x \geq 3 \end{cases}$

(J) $j(x) = \begin{cases} 3x + 4 & \text{if } x \leq -1 \\ 3^x + 4 & \text{if } x > -1 \end{cases}$

35. Let $f(x) = \begin{cases} 1 - 5x & \text{if } x < -5 \\ 3 - x^3 & \text{if } -5 \leq x < -2 \\ 5 - x^2 & \text{if } x \geq -2 \end{cases}$. Find $f(-2)$.
- (A) -5 (B) 1 (C) 9 (D) 11

CHALLENGE AND EXTEND

The *greatest integer function* returns the greatest integer less than or equal to a given number. The greatest integer function is written $f(x) = \lfloor x \rfloor$ and is often written as $\text{int}(x)$ on graphing calculators. For example, if hamburgers cost \$1.79 each, the function $f(x) = \lfloor \frac{x}{1.79} \rfloor$ would return the number of hamburgers you could buy for x dollars.

36. Write a function for the number of orders of fries that can be bought with x dollars if an order of fries costs \$1.29. Then use your function to find the number of orders of fries that you can buy with \$10.

The *least integer function* returns the least integer greater than or equal to a given number. The least integer function is written $f(x) = \lceil x \rceil$. For example, $f(2.9) = \lceil 2.9 \rceil = 3$.

37. At a parking garage, parking costs \$4 for up to 1 hour. After that, it costs \$1.50 for each additional hour or fraction thereof. Write a function to represent the cost of parking for x hours. Then use the function to find the cost of parking for 5 hours and 23 minutes.

SPIRAL REVIEW



38. **Geometry** There is a linear relationship between the number of sides in a regular polygon and the number of degrees in that polygon, as shown in the table. Write a function to represent the relationship. (Lesson 2-4)

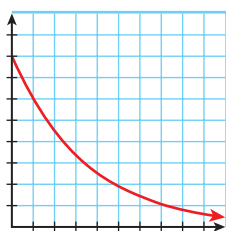
Sides	3	4	5	6	8
Sum of Interior Angles (°)	180	360	540	720	1080

Identify the asymptotes, domain, and range of each function. (Lesson 8-4)

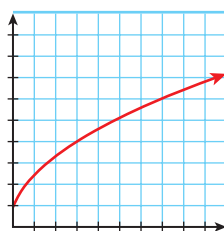
39. $f(x) = \frac{4}{x-1} - 3$ 40. $f(x) = \frac{3}{x+2} + 1$ 41. $f(x) = \frac{5}{x-3} + 1$

Match each situation with one of the following graphs. (Lesson 9-1)

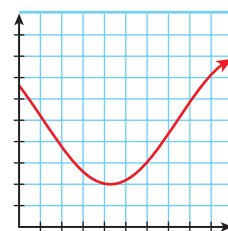
A.



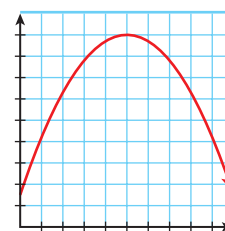
B.



C.



D.



42. A company releases a product without advertisement, and the profit drops. Then the company advertises, and the profit increases.
43. The value of a computer declines over time.
44. The sales for an ice cream store are low in winter, high in spring and fall, and extremely high in summer.
45. The temperature rises steadily from 12:00 P.M. to 5:00 P.M.



Graph Piecewise Functions

You can graph piecewise functions on a graphing calculator by using logical tests to restrict the domain for each piece of the function.

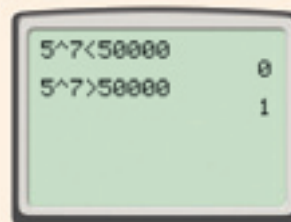
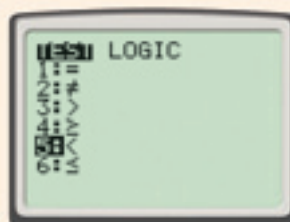
Use with Lesson 9-2

Activity 1

A graphing calculator can determine whether mathematical statements, such as $5 > 3$, are true. You can enter these statements by using the **TEST** menu. The calculator returns a value of 1 if the statement is true and a value of 0 if the statement is false.

Determine whether the statement 5^7 is greater than or less than 50,000.

Enter the left side of the first expression. Access the **TEST** menu (as shown) by pressing **2nd** **MATH**. Choose the less-than symbol (**5:<**), and then complete the expression. Enter the second expression, substituting the greater-than symbol (**3:>**).



The first expression returns a value of 0, so it is false. The second expression returns a value of 1, so it is true. The expression 5^7 is greater than 50,000.

Try This

Use logical tests to determine whether each statement is true or false.

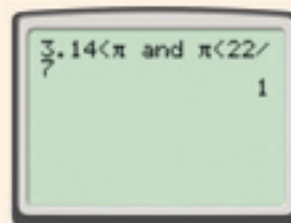
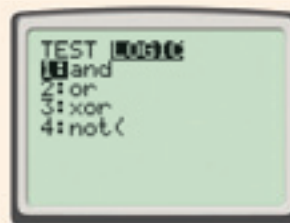
- $4 - 3 = 3 - 4$
- $(-6)^4 \geq 1000$
- $\frac{3}{16} < 0.1875$
- $\frac{3}{16} > 0.1875$
- Draw a Conclusion** What conclusion can you make about $\frac{3}{16}$ and 0.1875 based on the answers to Problems 3 and 4?

Activity 2

You can use the commands from the **LOGIC** submenu of the **TEST** menu to create compound logical tests.

Determine whether the statement $3.14 < \pi < \frac{22}{7}$ is true.

Recall that the compound inequality $3.14 < \pi < \frac{22}{7}$ can be written as $3.14 < \pi$ and $\pi < \frac{22}{7}$. Enter the first expression, and then access the **LOGIC** submenu by pressing **2nd** **MATH**. Choose **1: and**, and then enter the second expression.



Because the statement returns a value of 1, the statement is true:
 $3.14 < \pi < \frac{22}{7}$. ✓

Try This

Use logical tests to determine whether each statement is true or false.

6. $-2^2 < -2 < (-2)^2$

7. $(-2)^1 < (-2)^2 < (-2)^3$

8. $\sqrt{2} < 2 < 2^2$

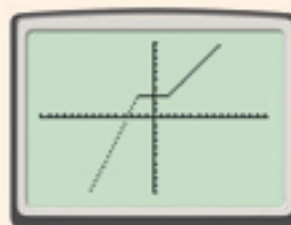
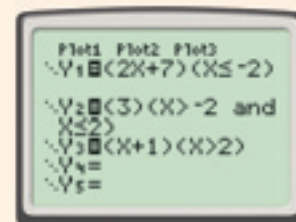
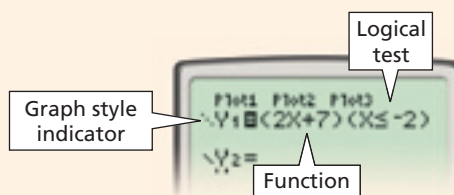
9. $\sqrt{\frac{1}{2}} < \frac{1}{2} < \left(\frac{1}{2}\right)^2$

10. **Make a Conjecture** What conjecture can you make about the relationship between a number, its square, and its square root if the number is greater than 1? What if the number is between 0 and 1?

Activity 3

Graph the piecewise function $f(x) = \begin{cases} 2x + 7 & \text{if } x \leq -2 \\ 3 & \text{if } -2 < x \leq 2 \\ x + 1 & \text{if } x > 2 \end{cases}$.

- When you want to graph any piecewise function, change the calculator to dot mode. Press **MODE**, and then select **Dot**. This will prevent the calculator from showing extraneous lines.
- Enter the first part of the rule, enclosed in parentheses, as **Y1**. If your calculator is not in dot mode, change the graph style so that three dots are shown. In a second set of parentheses, enter the logical test that will restrict the domain as indicated in the function rule.
- Use similar methods to enter the second part of the rule as **Y2**. Because the restriction is a compound inequality, you will have to use the **and** command from the **LOGIC** menu.
- Finally, enter the third part of the rule as **Y3**, and graph in the standard square window.
- The table shows the function values for each function over the restricted domain. Outside of the restricted domain, each function shows values of zero.



X	Y1	Y2
-2.5	0	0
-2	0	0
-1.5	0	0
-1	0	0
-0.5	0	0
0	0	0
0.5	0	0
1	0	0
1.5	0	0
2	0	0
2.5	0	0
3	0	0
X=-2		

Try This

Graph each piecewise function.

11. $g(x) = \begin{cases} x & \text{if } x < 0 \\ -x & \text{if } x \geq 0 \end{cases}$

12. $h(x) = \begin{cases} 2x + 8 & \text{if } x \leq -2 \\ x^2 & \text{if } x > -2 \end{cases}$

13. $f(x) = \begin{cases} -3x & \text{if } x < 1 \\ x - 4 & \text{if } 1 \leq x < 5 \\ -\frac{1}{2}x + \frac{7}{2} & \text{if } x \geq 5 \end{cases}$

14. **Critical Thinking** Explain how multiplying the function by a logical test value visually restricts the domain of the function on your graph.
15. **Critical Thinking** Explain why you should use dot mode instead of connected mode when graphing piecewise functions.



9-3

Transforming Functions

Objectives

Transform functions.

Recognize transformations of functions.

Why learn this?

Transformations can be used to describe changes in college tuition fees. (See Example 4.)

In previous lessons, you learned how to transform several types of functions. You can transform piecewise functions by applying transformations to each piece independently. Recall the rules for transforming functions given in the table.



"If you miss a payment, we show up and embarrass you in front of your friends."

© Cartoon Stock



Transformations of $f(x)$	
Horizontal Translation	Vertical Translation
$f(x) \rightarrow f(x - h)$ left for $h < 0$ right for $h > 0$	$f(x) \rightarrow f(x) + k$ down for $k < 0$ up for $k > 0$
Reflection Across y-axis	Reflection Across x-axis
$f(x) \rightarrow f(-x)$ The graph is reflected across the y-axis.	$f(x) \rightarrow -f(x)$ The graph is reflected across the x-axis.
Horizontal Stretch/Compression	Vertical Stretch/Compression
$f(x) \rightarrow f\left(\frac{1}{b}x\right)$ stretch for $b > 1$ compression for $0 < b < 1$	$f(x) \rightarrow af(x)$ stretch for $a > 1$ compression for $0 < a < 1$

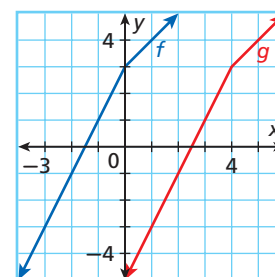
EXAMPLE 1 Transforming Piecewise Functions

Given $f(x) = \begin{cases} x + 3 & \text{if } x > 0 \\ 2x + 3 & \text{if } x \leq 0 \end{cases}$, write the rule for $g(x)$, a horizontal translation of $f(x)$ 4 units right.

Each piece of $f(x)$ must be shifted 4 units right. Replace every x in the function with $(x - 4)$, and simplify.

$$\begin{aligned}
 g(x) = f(x - 4) &= \begin{cases} (x - 4) + 3 & \text{if } (x - 4) > 0 \\ 2(x - 4) + 3 & \text{if } (x - 4) \leq 0 \end{cases} \\
 &= \begin{cases} x - 1 & \text{if } x > 4 \\ 2x - 5 & \text{if } x \leq 4 \end{cases}
 \end{aligned}$$

Check Graph both functions to support your answer.



Caution!

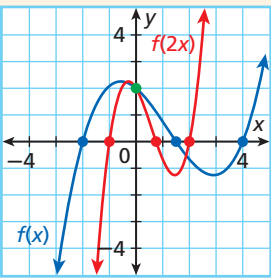
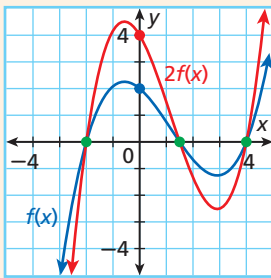
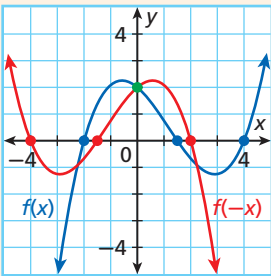
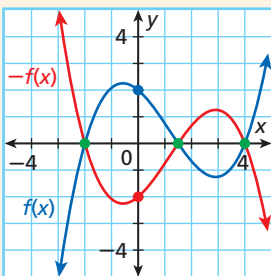
Horizontal translations change both the rules and the intervals of piecewise functions. Vertical translations change only the rules.



1. Given $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x - 3 & \text{if } x > 0 \end{cases}$, write the rule for $g(x)$,
a horizontal stretch of $f(x)$ by a factor of 2.

When functions are transformed, the intercepts may or may not change. By identifying the transformations, you can determine the intercepts, which can help you graph a transformed function.



Effects of Transformations on Intercepts of $f(x)$	
Horizontal Stretch or Compression by a Factor of b	Vertical Stretch or Compression by a Factor of a
 x-intercepts are multiplied by b. y-intercept stays the same.	 x-intercepts stay the same. y-intercept is multiplied by a.
Reflection Across y-axis	Reflection Across x-axis
 x-intercepts are negated. y-intercept stays the same.	 x-intercepts stay the same. y-intercept is negated.

EXAMPLE 2 Identifying Intercepts

Identify the x - and y -intercepts of $f(x)$. Without graphing $g(x)$, identify its x - and y -intercepts.

A $f(x) = \frac{1}{2}x - 3$ and $g(x) = 3f(x)$

Find the intercepts of the original function.

$$f(0) = \frac{1}{2}(0) - 3 = -3 \qquad 0 = \frac{1}{2}x - 3$$

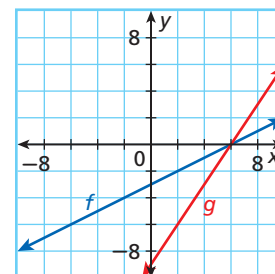
$$f(0) = -3 \qquad 6 = x$$

The y -intercept is -3 , and the x -intercept is 6 .

Note that $g(x)$ is a vertical stretch of $f(x)$ by a factor of 3 . So the x -intercept of $g(x)$ is also 6 .

The y -intercept is $3(-3)$, or -9 .

Check A graph supports your answer.



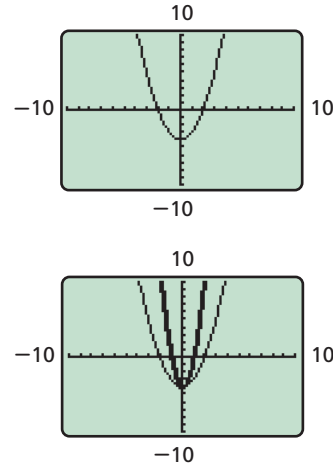
Identify the x - and y -intercepts of $f(x)$. Without graphing $g(x)$, identify its x - and y -intercepts.

B $f(x) = x^2 - 4$ and $g(x) = f(2x)$

From the graph of $f(x)$, the y -intercept is -4 and the x -intercepts are -2 and 2 .

Note that $g(x)$ is a horizontal compression by a factor of $\frac{1}{2}$. So the x -intercepts of $g(x)$ will be $\frac{1}{2}(-2)$ and $\frac{1}{2}(2)$, or -1 and 1 . The y -intercept is unchanged at -4 .

Check A graph supports your answer.



Identify the x - and y -intercepts of $f(x)$. Without graphing $g(x)$, identify its x - and y -intercepts.

2a. $f(x) = \frac{2}{3}x + 4$ and $g(x) = -f(x)$

2b. $f(x) = x^2 - 9$ and $g(x) = \frac{1}{3}f(x)$

EXAMPLE 3 Combining Transformations

Given $f(x) = -\frac{2}{3}x^2 + 6$ and $g(x) = f\left(\frac{3}{2}x\right) + 4$, graph $g(x)$.

Step 1 Graph $f(x)$. The graph of $f(x)$ has y -intercept $(0, 6)$ and x -intercepts $(-3, 0)$ and $(3, 0)$.

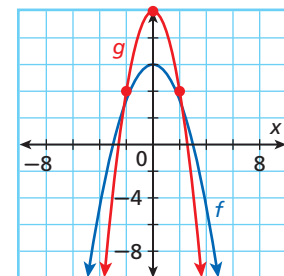
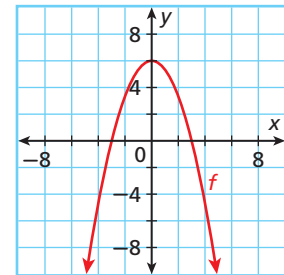
Step 2 Analyze each transformation one at a time.

The first transformation is a horizontal compression by a factor of $\frac{2}{3}$. After the horizontal compression, the x -intercepts will be -2 and 2 , but the y -intercept will remain 6 .

The second transformation is a vertical translation of 4 units up. Use a table to shift each identified point up 4 units.

Intercept Points	$(-2, 0)$	$(2, 0)$	$(0, 6)$
Shifted	$(-2, 4)$	$(2, 4)$	$(0, 10)$

Step 3 Graph the final result.



Remember!

The factor for horizontal stretches and compressions is the reciprocal of the coefficient in the equation.

$$\frac{1}{\frac{3}{2}} = \frac{2}{3}$$



3. Given $f(x) = 2^x - 4$ and $g(x) = -\frac{1}{2}f(x)$, graph $g(x)$.

EXAMPLE 4**Problem-Solving Application**

A college charges different fees according to the number of credit hours in which students have enrolled. The fee scale is modeled by the piecewise function below, where x is the number of credit hours.

$$f(x) = \begin{cases} 110x & \text{if } 0 < x < 12 \\ 1320 & \text{if } 12 \leq x \leq 18 \\ 150(x - 18) + 1320 & \text{if } x > 18 \end{cases}$$

The college plans to increase all fees by 10% for the fall semester. In the spring semester, the college plans to add an administrative fee of \$75 to each enrollment. Write the rule for the fee function for the spring semester.

1 Understand the Problem

The new fee function will include two changes, a 10% increase and an additional fee of \$75. The 10% increase is equivalent to multiplying all of the parts of the function by 110%, or 1.1. This will be a vertical stretch by a factor of 1.1. The administrative fee will be a vertical translation of 75 units up.

2 Make a Plan

Perform each transformation, one at a time, and then write the new rule.

3 Solve

First find the fees for the fall semester.

$$\begin{aligned} f_{\text{fall}}(x) &= (1.1)f(x) = \begin{cases} (1.1)110x & \text{if } 0 < x < 12 \\ (1.1)1320 & \text{if } 12 \leq x \leq 18 \\ (1.1)[150(x - 18) + 1320] & \text{if } x > 18 \end{cases} \quad \text{Multiply all parts by 1.1.} \\ &= \begin{cases} 121x & \text{if } 0 < x < 12 \\ 1452 & \text{if } 12 \leq x \leq 18 \\ 165(x - 18) + 1452 & \text{if } x > 18 \end{cases} \end{aligned}$$

Then find the fees for the spring semester.

$$\begin{aligned} f_{\text{spring}}(x) &= f_{\text{fall}}(x) + 75 = \begin{cases} 121x + 75 & \text{if } 0 < x < 12 \\ 1452 + 75 & \text{if } 12 \leq x \leq 18 \\ 165(x - 18) + 1452 + 75 & \text{if } x > 18 \end{cases} \\ &= \begin{cases} 121x + 75 & \text{if } 0 < x < 12 \\ 1527 & \text{if } 12 \leq x \leq 18 \\ 165(x - 18) + 1527 & \text{if } x > 18 \end{cases} \end{aligned}$$

4 Look Back

Check your answer by trying a few values. For 20 hours, the original fee would have been \$1620. A 10% increase plus a \$75 fee would amount to \$1857. Evaluate the function for $x = 20$ to check.

$$f_{\text{spring}}(20) = 165(20 - 18) + 1527 = 1857 \quad \checkmark$$

Continue by checking each piece of the function.



4. A movie theater charges \$5 for children under 12 and \$7.50 for anyone 12 and over. The theater decides to increase its prices by 20%. It charges an additional \$0.50 fee for online ticket purchases. Write a function for the online ticket prices.

THINK AND DISCUSS

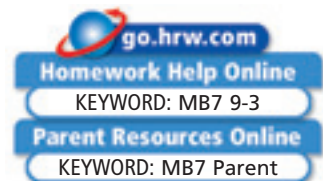
1. Identify the transformations that leave the y -intercept unchanged.
2. Explain why the point $(0, 0)$ is unchanged under any stretch or compression.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Identify the effects of each transformation on the intercepts.



Transformation	x -intercepts	y -intercept
Horizontal stretch or compression by a factor of b		
Vertical stretch or compression by a factor of a		
Reflection across y -axis		
Reflection across x -axis		

9-3

Exercises



GUIDED PRACTICE

SEE EXAMPLE 1
p. 672

Given $f(x) = \begin{cases} x - 3 & \text{if } x \leq 0 \\ 4x & \text{if } x > 0 \end{cases}$, write the rule for each function.

1. $g(x)$, a horizontal translation of $f(x)$ 6 units left
2. $h(x)$, a horizontal compression by a factor of $\frac{1}{4}$

SEE EXAMPLE 2
p. 673

Identify the x - and y -intercepts of $f(x)$. Without graphing $g(x)$, identify its x - and y -intercepts.

3. $f(x) = 4x + 12$ and $g(x) = \frac{1}{6}f(x)$
4. $f(x) = -x^2 + 16$ and $g(x) = f(4x)$

SEE EXAMPLE 3
p. 674

Given $f(x)$, graph $g(x)$.

5. $f(x) = -x^2 + 1$ and $g(x) = f(2x) - 1$
6. $f(x) = |x - 1| - 2$ and $g(x) = -2f(x)$

SEE EXAMPLE 4
p. 675

7. **Taxes** The state income tax in Connecticut is modeled by the function

$$T(x) = \begin{cases} 0.02x & \text{if } 0 < x \leq 10,000 \\ 0.05x & \text{if } x > 10,000 \end{cases}, \text{ where } x \text{ is income in dollars. Suppose that}$$

Connecticut decided to increase its tax rates by 20% and add a filing fee of \$100 dollars. Write a function for the new state income tax.

PRACTICE AND PROBLEM SOLVING

Given $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 4x & \text{if } x \geq 1 \end{cases}$, write the rule for each function.

8. $g(x)$, a vertical compression of $f(x)$ by a factor of $\frac{1}{4}$
9. $h(x)$, a horizontal stretch by a factor of 2
10. $p(x)$, a vertical translation 3 units down

Independent Practice

For Exercises	See Example
8–10	1
11–16	2
17–18	3
19	4

Extra Practice

Skills Practice p. S20

Application Practice p. S40

Identify the x - and y -intercepts of $f(x)$. Without graphing $g(x)$, identify its x - and y -intercepts.

11. $f(x) = -\frac{3}{2}x + 9$ and $g(x) = \frac{2}{3}f(x)$

12. $f(x) = x^2 - 25$ and $g(x) = f\left(\frac{5}{3}x\right)$

13. $f(x) = -\frac{2}{5}x + 2$ and $g(x) = f(2x)$

14. $f(x) = x^2 - 3x - 4$ and $g(x) = -f\left(\frac{1}{3}x\right)$

15. $f(x) = 3^x - 1$ and $g(x) = 2f(x) - 4$

16. $f(x) = x^3 + 8$ and $g(x) = f\left(-\frac{1}{2}x\right)$

Given $f(x)$, graph $g(x)$.

17. $f(x) = \frac{1}{2}x + 4$ and $g(x) = 3f(-x)$

18. $f(x) = \left(\frac{1}{2}\right)^x - 2$ and $g(x) = -f(2x)$

19. **Business** The amount that a caterer charges to cater a party for n people is given by the function $C(n) = \begin{cases} 18n & \text{if } n \leq 50 \\ 400 + 10n & \text{if } n > 50 \end{cases}$.

- During a sale, the caterer reduces the amount charged by 10%. Find the function for how much the caterer will charge during the sale.
- If the caterer then decides to take an additional \$2 off per person, find the function for how much the caterer will charge.

20. **Safety** Speeding fines in Washington, D.C., are shown in the table.

- Write a function to represent speeding fines.
- If the speeding offense occurs in a school zone, the city adds a fine of \$50. Write a function for the increased fines in a school zone.
- What if...?** The city is considering increasing speeding fines by 15%. Write a new function for the increased speeding fines.

Speeding Fines in Washington, D.C.	
Speed Over Limit (mi/h)	Fine (\$)
1–10	30
11–15	50
16–20	100
21–25	150
26–30	200

21. **Critical Thinking** Suppose that the graph of $f(x)$ has n x -intercepts.

- How many x -intercepts does the graph of $bf(ax)$ have? Explain.
- Explain why you cannot tell how many x -intercepts the graph of $f(x - h) + k$ has.

22. **Money** A credit card company charges a person taking a cash advance on its credit card at an ATM a \$6 transaction fee if \$200 or less is withdrawn. For amounts over \$200, the transaction fee is 3% of the amount withdrawn.

- Write a function for the transaction fee to withdraw x dollars.
- Suppose that the company wants to increase fees by 15% in order to reflect increased costs. Adjust your function to include the increase.



Safety

In some cities, automatic cameras are used to identify and photograph vehicles that are speeding. The first stationary speed camera in Washington, D.C., detected more than 10,000 speeding drivers in its first 15 days of operation.

MULTI-STEP
TEST PREP



23. This problem will prepare you for the Multi-Step Test Prep on page 680.

At a party, the host whispers a phrase into the ear of a guest who then whispers the phrase into the ear of the person standing next to him. The process is repeated down the line until the last person says the phrase out loud. The time in seconds for the phrase to move through the line is modeled by $T(n) = \begin{cases} 3.5n & \text{if } n \leq 8 \\ 4.5n - 8 & \text{if } n > 8 \end{cases}$, where n is the number of guests in the line.

- The second time that the game is played, each person's reaction time is 20% faster. Write a new function to model this situation.
- Describe the effect of this improvement on the graph of $T(n)$.



24. **Technology** Morphing is a computerized technique for making one picture turn into another picture. Morphing is created by transforming specific points from one location to another.

- Graph the functions $f(x) = \begin{cases} \frac{1}{2}x + 4 & \text{for } 1 \leq x \leq 2 \\ -x^2 + 6x - 7 & \text{for } 2 \leq x \leq 4 \end{cases}$ and $h(x) = \begin{cases} -\frac{1}{2}x + 7 & \text{for } 4 \leq x \leq 5 \end{cases}$ on the same coordinate grid.
- Graph the transformed functions $f_{\text{new}}(x) = -f(x) + 8$, $g_{\text{new}}(x) = -g(x) + 3$, and $h_{\text{new}}(x) = -h(x) + 8$.
- Describe the morph that you created.

For each function, give the new function rule after the given transformation.

- $f(x) = \begin{cases} 2^x - 1 & \text{if } x \leq -3 \\ -5x + 3 & \text{if } x > -3 \end{cases}$ after a translation of 7 units down
- $f(x) = \begin{cases} 3x^2 & \text{if } x < 1 \\ -2x + 4 & \text{if } x \geq 1 \end{cases}$ after a vertical stretch by a factor of 5

27. **Food** A farmers' market sells fruits and vegetables at a flat rate with discounts for larger purchases.

- Sketch a graph of the cost of 0 to 10 pounds of produce.
- Write a piecewise function for the cost of x pounds of produce.
- What if...?** During a sale, the market offers a buy-one-get-one-free sale. Graph the new cost function and describe the transformation from the original function.



- Business** A company's profit model is given by $P(n) = -0.002n^2 + 19n - 9000$, where $P(n)$ is the profit in dollars and n is the number of items produced. Based on some new data, the profit model for next year is predicted to be $R(n) = P(0.8n)$.
 - How will the change affect the number of items the company should produce to maximize its profit?
 - Find the number of items the company should produce to maximize profit under the new model.
- Critical Thinking** A linear function has an x -intercept equal to 2 and a y -intercept equal to 3. The function is stretched vertically by a factor of 2, then translated 3 units down, and then stretched horizontally by a factor of 2. What are the new intercepts?
- Critical Thinking** Why does a vertical translation not affect the domain of a function but a horizontal translation might? Explain.
-  **Write About It** Can a graph that is not continuous be transformed into a continuous graph by using stretches and compressions only? Explain.

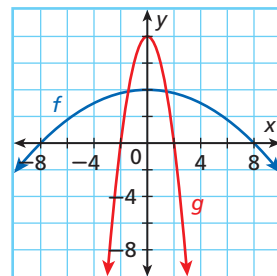
32. For the graphs shown, which of the following is $g(x)$?

(A) $g(x) = 2f\left(\frac{1}{4}x\right)$

(C) $g(x) = 2f(4x)$

(B) $g(x) = \frac{1}{2}f\left(\frac{1}{4}x\right)$

(D) $g(x) = \frac{1}{2}f(4x)$



33. Suppose that $f(x) = \begin{cases} 2x & x > 8 \\ x^2 & x \leq 8 \end{cases}$.

Which of the following is $g(x) = f(4x)$?

(F) $g(x) = \begin{cases} \frac{x}{2} & \text{if } x > 2 \\ \frac{x^2}{16} & \text{if } x \leq 2 \end{cases}$

(H) $g(x) = \begin{cases} 8x & \text{if } x > 32 \\ 4x^2 & \text{if } x \leq 32 \end{cases}$

(G) $g(x) = \begin{cases} \frac{x}{2} & \text{if } x > 8 \\ \frac{x^2}{16} & \text{if } x \leq 8 \end{cases}$

(J) $g(x) = \begin{cases} 8x & \text{if } x > 2 \\ 16x^2 & \text{if } x \leq 2 \end{cases}$

34. The y -intercept of $g(x) = \frac{3}{5}f(5x)$ is 15. Which of the following is the y -intercept of $f(x)$?

(A) 3

(B) 9

(C) 25

(D) 75

CHALLENGE AND EXTEND



35. **Geometry** Consider the function $f(x) = \begin{cases} \frac{2}{3}x + 4 & \text{if } x < 0 \\ -\frac{1}{2}x + 4 & \text{if } x \geq 0 \end{cases}$.

- Graph the function, and find its intercepts. Then find the area bounded by the function and the x -axis.
- Graph the transformation $g(x) = 4f(2x)$. Find the area bounded by $g(x)$ and the x -axis.
- Write a function $h(x)$ that creates an area of 7 square units.

36. Consider the functions $f(x) = 2x^3 - 3x^2 - 11x + 6$, $g(x) = 3f\left(\frac{1}{2}x\right)$, and $h(x) = -g\left(\frac{1}{2}x\right)$.

- Find the x - and y -intercepts of $g(x)$.
- Find the x - and y -intercepts of $h(x)$.

SPIRAL REVIEW

37. **Geology** An earthquake map claims that about 43% of the earthquakes in the United States between 1999 and 2002 occurred in California. There were 973 earthquakes between 1999 and 2002 in the United States. About how many earthquakes occurred in California? (*Lesson 2-2*)

Find the maximum or minimum value of each function. Then state the domain and range of the function. (*Lesson 5-2*)

38. $f(x) = 4x^2 - 2x + 8$

39. $g(x) = -3x^2 + 6x - 9$

Evaluate each piecewise function for $x = -4$, $x = 0$, and $x = 5$. (*Lesson 9-2*)

40. $f(x) = \begin{cases} 3 & \text{if } x < 1 \\ x^2 - 4 & \text{if } x \geq 1 \end{cases}$

41. $f(x) = \begin{cases} 5 - 2x & \text{if } x < -3 \\ 4 + x & \text{if } x \geq -3 \end{cases}$

MULTI-STEP TEST PREP



Functions and Their Graphs

Hands Around the World Imagine a human chain of people holding hands. Assume that each person stands with his or her arms fully outstretched.

1. Suppose that the chain could go all the way around the planet. Then the chain's length would be equal to the circumference of the earth at the equator (about 24,000 miles). Assuming that the average adult arm span is 6 feet, how many people would it take to make this human chain?
2. At the word "go!" the first person in the chain squeezes the hand of the second person, who in turn immediately squeezes the hand of the third person, and so on. Given that it takes 20 seconds for the 60th person to react to having his or her hand squeezed, how many hours would it take the signal to travel all the way around the world?
3. Sketch a graph of the distance in feet that the signal travels in the span of 0 to 1000 seconds. Identify the slope of the line.
4. Researchers at the University of British Columbia have measured muscle reaction times of 0.1 second for Olympic sprinters. If the human chain consisted entirely of Olympic sprinters, how long would it take for the signal to travel all the way around the world?
5. Create a graph of the distance the signal travels in the chain of Olympic sprinters. How does the slope compare to the graph in Problem 3?
6. Suppose that the first half of a human chain is formed by 500 Olympic sprinters and the second half is formed by 500 people whose reaction time is 0.9 second. Write and graph a piecewise function that describes the distance traveled by the signal as a function of time.



Quiz for Lessons 9-1 Through 9-3



9-1 Multiple Representations of Functions

- Amanda must read a 294-page book for her history class over the next week. Amanda has found that she can read 42 pages in an hour. Create a table, a graph, and an equation to represent the number of pages that Amanda has left to read with relation to time.
- The height of a rocket at different times after it was fired is shown in the table.

Time (s)	0	1	2	3	4	5
Height (m)	50.0	65.1	70.4	65.9	51.5	27.5

- Find an appropriate model for the height of the rocket.
- Find the maximum height of the rocket.
- How long will the rocket stay in the air?



9-2 Piecewise Functions

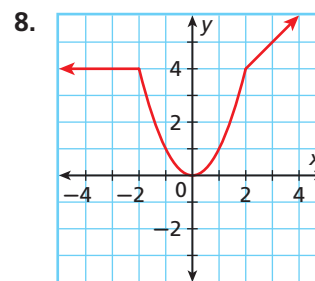
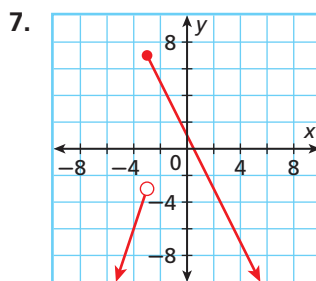
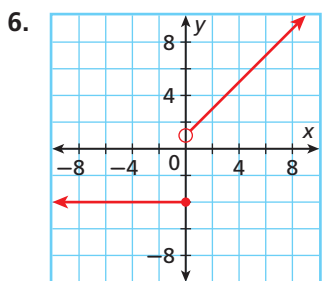
Graph each function.

3. $f(x) = \begin{cases} 3 & \text{if } x < 0 \\ 2x + 3 & \text{if } x \geq 0 \end{cases}$

4. $h(x) = \begin{cases} -x + 1 & \text{if } x < -3 \\ -x & \text{if } -3 \leq x < 1 \\ -x - 1 & \text{if } x \geq 1 \end{cases}$

- The cost of renting a mountain bike is \$25 for the first 3 hours and \$5 for each additional hour. Sketch a graph of the cost of renting a mountain bike for 0 to 8 hours. Then write a piecewise function for the graph.

Write a piecewise function for each graph.



9-3 Transforming Functions

Identify the x - and y -intercepts of $f(x)$. Without graphing $g(x)$, identify its x - and y -intercepts.

9. $f(x) = 2x - 2$ and $g(x) = -f\left(\frac{1}{2}x\right)$

10. $f(x) = x^2 - 4$ and $g(x) = 2f(x)$

Given $f(x)$, graph $g(x)$.

11. $f(x) = |x| - 3$ and $g(x) = 2f(x) + 3$

12. $f(x) = x^2 + 1$ and $g(x) = -3f(x)$

Operations with Functions

Objectives

Add, subtract, multiply, and divide functions.

Write and evaluate composite functions.

Vocabulary

composition of functions

Who uses this?

Importers can use function operations to determine the costs of items that are purchased in foreign currencies. (See Example 5.)

You can perform operations on functions in much the same way that you perform operations on numbers or expressions. You can add, subtract, multiply, or divide functions by operating on their rules.



Notation for Function Operations	
Operation	Notation
Addition	$(f + g)(x) = f(x) + g(x)$
Subtraction	$(f - g)(x) = f(x) - g(x)$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ where } g(x) \neq 0$

EXAMPLE 1

Adding and Subtracting Functions

Given $f(x) = 2x^2 + 4x - 6$ and $g(x) = 2x - 2$, find each function.

A $(f + g)(x)$

$$\begin{aligned} (f + g)(x) &= f(x) + g(x) \\ &= (2x^2 + 4x - 6) + (2x - 2) && \text{Substitute function rules.} \\ &= 2x^2 + 6x - 8 && \text{Combine like terms.} \end{aligned}$$

B $(f - g)(x)$

$$\begin{aligned} (f - g)(x) &= f(x) - g(x) \\ &= (2x^2 + 4x - 6) - (2x - 2) && \text{Substitute function rules.} \\ &= 2x^2 + 4x - 6 - 2x + 2 && \text{Distributive Property} \\ &= 2x^2 + 2x - 4 && \text{Combine like terms.} \end{aligned}$$



Given $f(x) = 5x - 6$ and $g(x) = x^2 - 5x + 6$, find each function.

1a. $(f + g)(x)$

1b. $(f - g)(x)$

When you divide functions, be sure to note any domain restrictions that may arise.

EXAMPLE 2 Multiplying and Dividing Functions

Given $f(x) = 2x^2 + 4x - 6$ and $g(x) = 2x - 2$, find each function.

A $(gf)(x)$

$$\begin{aligned}(gf)(x) &= g(x) \cdot f(x) \\&= (2x - 2)(2x^2 + 4x - 6) && \text{Substitute function rules.} \\&= 2x(2x^2 + 4x - 6) - 2(2x^2 + 4x - 6) && \text{Distributive Property} \\&= 4x^3 + 8x^2 - 12x - 4x^2 - 8x + 12 && \text{Multiply.} \\&= 4x^3 + 4x^2 - 20x + 12 && \text{Combine like terms.}\end{aligned}$$

B $\left(\frac{f}{g}\right)(x)$

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} \\&= \frac{2x^2 + 4x - 6}{2x - 2} && \text{Set up the division as a rational expression.} \\&= \frac{2(x - 1)(x + 3)}{2(x - 1)} && \text{Factor completely. Note that } x \neq 1. \\&= \frac{\cancel{2}(x - \cancel{1})(x + 3)}{\cancel{2}(x - \cancel{1})} && \text{Divide out common factors.} \\&= x + 3, \text{ where } x \neq 1 && \text{Simplify.}\end{aligned}$$



Given $f(x) = x + 2$ and $g(x) = x^2 - 4$, find each function.

2a. $(fg)(x)$

2b. $\left(\frac{g}{f}\right)(x)$

Another function operation uses the output from one function as the input for a second function. This operation is called the **composition of functions**.



Composition of Functions

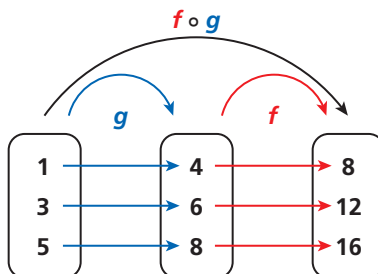
The composition of functions f and g is notated

$$(f \circ g)(x) = f(g(x)).$$

The domain of $(f \circ g)(x)$ is all values of x in the domain of g such that $g(x)$ is in the domain of f .

Reading Math

The composition $(f \circ g)(x)$ or $f(g(x))$ is read "f of g of x."



To find $(f \circ g)(1)$, first find $g(1)$.

$$g(1) = 4$$

Then use 4 as the input into f :

$$f(4) = 8$$

$$\text{So } (f \circ g)(1) = f(g(1)) = 8.$$

The order of function operations is the same as the order of operations for numbers and expressions. To find $f(g(3))$, evaluate $g(3)$ first and then substitute the result into f .

EXAMPLE 3 Evaluating Composite Functions**Caution!**

Be careful not to confuse the notation for multiplication of functions with composition. $fg(x) \neq f(g(x))$

Given $f(x) = 3x + 1$ and $g(x) = x^3$, find each value.

A $f(g(2))$

Step 1 Find $g(2)$.

$$g(2) = 2^3 \quad g(x) = x^3$$

$$= 8$$

Step 2 Find $f(8)$.

$$f(8) = 3(8) + 1 \quad f(x) = 3x + 1$$

$$= 25$$

So $f(g(2)) = 25$.

B $g(f(2))$

Step 1 Find $f(2)$.

$$f(2) = 3(2) + 1 \quad f(x) = 3(x) + 1$$

$$= 7$$

Step 2 Find $g(7)$.

$$g(7) = 7^3 \quad g(x) = x^3$$

$$= 343$$

So $g(f(2)) = 343$.



Given $f(x) = 2x - 3$ and $g(x) = x^2$, find each value.

3a. $f(g(3))$

3b. $g(f(3))$

You can use algebraic expressions as well as numbers as inputs into functions. To find a rule for $f(g(x))$, substitute the rule for g into f .

EXAMPLE 4 Writing Composite Functions

Given $f(x) = 5x + 2$ and $g(x) = \frac{2}{x-1}$, write each composite function. State the domain of each.

A $f(g(x))$

$$f(g(x)) = f\left(\frac{2}{x-1}\right)$$

Substitute the rule for g into f .

$$= 5\left(\frac{2}{x-1}\right) + 2$$

Use the rule for f . Note that $x \neq 1$.

$$= \frac{10}{x-1} + 2, x \neq 1$$

Simplify.

The domain of $f(g(x))$ is $x \neq 1$ or $\{x \mid x \neq 1\}$ because $g(1)$ is undefined.

B $g(f(x))$

$$g(f(x)) = g(5x + 2)$$

Substitute the rule for f into g .

$$= \frac{2}{(5x + 2) - 1}$$

Use the rule for g .

$$= \frac{2}{5x + 1}, x \neq -\frac{1}{5}$$

Simplify. Note that $x \neq -\frac{1}{5}$.

The domain of $g(f(x))$ is $x \neq -\frac{1}{5}$ or $\{x \mid x \neq -\frac{1}{5}\}$ because $f\left(-\frac{1}{5}\right) = 1$ and $g(1)$ is undefined.



Given $f(x) = 3x - 4$ and $g(x) = \sqrt{x} + 2$, write each composite function. State the domain of each.

4a. $f(g(x))$

4b. $g(f(x))$

Composite functions can be used to simplify a series of functions.

EXAMPLE 5 Business Application

Lisa imports scooters from Italy. The cost of the scooters is given in euros. The total cost of each scooter includes a 10% service charge and 75 euros for shipping.



- A** Write a composite function to represent the total cost of the scooter in dollars if the cost of the item is c euros.

Step 1 Write a function for the total cost in euros.

$$\begin{aligned} E(c) &= c + 0.1c + 75 \\ &= 1.1c + 75 \end{aligned}$$

Step 2 Write a function for the cost in dollars based on the cost in euros.

$$D(c) = \frac{c}{0.77} \quad \text{Use the exchange rate table.}$$

Step 3 Find the composition $D(E(c))$.

$$\begin{aligned} D(E(c)) &= \frac{E(c)}{0.77} && \text{Substitute } E(c) \text{ for } c. \\ &= \frac{1.1c + 75}{0.77} && \text{Replace } E(c) \text{ with its rule.} \end{aligned}$$

- B** Find the cost of the scooter in dollars if it costs 1200 euros.

Evaluate the composite function for $c = 1200$.

$$\begin{aligned} D(E(1200)) &= \frac{1.1(1200) + 75}{0.77} \\ &\approx 1811.69 \end{aligned}$$

The scooter would cost \$1811.69, including all charges.



During a sale, a music store is selling all drum kits for 20% off. Preferred customers also receive an additional 15% off.

- 5a.** Write a composite function to represent the final cost of a kit that originally cost c dollars.
- 5b.** Find the cost of a drum kit priced at \$248 that a preferred customer wants to buy.

THINK AND DISCUSS

1. Explain why $(f + g)x = (g + f)x$ for any functions f and g .
2. Find two functions such that $f(g(x)) = g(f(x))$.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Write the correct notation for each function operation.



Operation	Notation
Addition	
Subtraction	
Multiplication	
Division	
Composition	



GUIDED PRACTICE

1. **Vocabulary** How is the *composition of functions* different from the other function operations?

SEE EXAMPLE 1 Given $f(x) = 8x + 13$ and $g(x) = x^2 - 5x$, find each function.

p. 682

2. $(f + g)(x)$

3. $(f - g)(x)$

4. $(g - f)(x)$

SEE EXAMPLE 2 Given $f(x) = 2x^2 + 2x$ and $g(x) = x + 1$, find each function.

p. 683

5. $(fg)(x)$

6. $\left(\frac{f}{g}\right)(x)$

7. $\left(\frac{g}{f}\right)(x)$

SEE EXAMPLE 3 Given $f(x) = 3x^2$ and $g(x) = 7 - x$, find each value.

p. 684

8. $f(g(5))$

9. $g(f(5))$

10. $f(g(-2))$

SEE EXAMPLE 4 Given $f(x) = x^2$, $g(x) = 2x - 3$, and $h(x) = \sqrt{x + 1}$, write each composite function. State the domain of each.

p. 684

11. $f(g(x))$

12. $g(f(x))$

13. $f(h(x))$

SEE EXAMPLE 5 14. **Consumer Economics** Ron is saving money for college. Each month he deposits 10% of his net income plus an additional \$50 into a savings account. His net income, after taxes have been taken out, is 80% of his gross income.

p. 685

- a. Write a composite function for the amount that Ron saves each month if his gross income is g .
- b. Find the amount that Ron saves in a month when his gross income is \$2400.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15–18	1
19–23	2
24–29	3
30–32	4
33	5

Extra Practice

Skills Practice p. S21

Application Practice p. S40

Given $f(x) = 2x^2 - 8$, $g(x) = x^2 + 5x + 6$, and $h(x) = 2x + 4$, find each function.

15. $(f + g)(x)$

16. $(f - g)(x)$

17. $(f + h)(x)$

18. $(g - h)(x)$

19. $(fg)(x)$

20. $\left(\frac{f}{g}\right)(x)$

21. $\left(\frac{h}{f}\right)(x)$

22. $(gh)(x)$

23. $\left(\frac{g}{h}\right)(x)$

Given $f(x) = 2\sqrt{x + 3}$ and $g(x) = -3x + 1$, find each value.

24. $f(g(1))$

25. $g(f(1))$

26. $f(g(4))$

27. $g(f(6))$

28. $f\left(g\left(\frac{4}{3}\right)\right)$

29. $g(f(97))$

Given $f(x) = 4x + 3$, $g(x) = \frac{x}{x+3}$, and $h(x) = -x^2 - 2$, write each composite function. State the domain of each.

30. $f(g(x))$

31. $g(f(x))$

32. $f(h(x))$

33. **Business** The cost of carpeting a room is \$4 per square yard plus \$100. Each square yard is equal to 9 square feet.

- a. Write a composite function for the cost of carpeting a room that covers x square feet.
- b. Find the square footage of a room that costs \$380 to carpet.



34. This problem will prepare you for the Multi-Step Test Prep on page 706.

When the air in a hot-air balloon is heated to 100°F , each cubic foot of air can lift about 7 g.

- Write a function $f(x)$ for the number of grams that can be lifted by a balloon containing $x \text{ ft}^3$ of air.
- The equation $g(x) = \frac{x}{453.6}$ converts x grams to pounds. Write a composite function for the number of pounds that can be lifted by a balloon containing $x \text{ ft}^3$ of air.
- Approximately how many cubic feet of air are needed to lift 1000 lb?



Earthquakes



Earthquakes are generally caused by movement along faults. Faults are cracks in Earth's crust, the largest of which occur at tectonic plate boundaries.

35. **Consumer Economics** Lanie has two coupons for a shoe store. One is for \$10 off, and the other is for 15% off.
- Write a function $f(p)$ for the final cost of an item of original price p if Lanie uses only the \$10-off coupon.
 - Write a function $g(p)$ for the final cost of an item of original price p if Lanie uses only the 15%-off coupon.
 - Find $f(g(p))$ and $g(f(p))$.
 - Which coupon should Lanie apply first? Explain.
 - Find the lowest price that Lanie could pay for a pair of shoes priced at \$49.

36. **Earthquakes** The shock waves created from an earthquake travel away from the epicenter at a rate of 9 km/s. As the radius of the circular waves increases, more and more area is affected by the earthquake.

- Find a function for the total area in square kilometers affected by the earthquake after t s.
- Geologists predict that the earthquake will be felt over an area of approximately $35,000 \text{ km}^2$. How long after the earthquake begins will this area be affected?

37. **Population** The population of Las Vegas, Nevada, can be approximated by the function $p(t) = 160,000 \cdot 1.05^t$, where t is the number of years since 1980. The number of doctors in Las Vegas can be approximated by the function $d(p) = 0.0044p$, where p is the population.

- Find a function for the number of doctors in Las Vegas as a function of the number of years t since 1980.
- Estimation** Estimate the number of doctors in Las Vegas in 2010.
- Approximately when will the number of doctors in Las Vegas exceed 5000?

38. **Critical Thinking** Given $f(x) = x$ and given any function $g(x)$, is $f(g(x))$ always equal to $g(f(x))$? Explain.

Use the tables to find each value.

39. $(g \circ f)(5)$ 40. $(f \circ g)(3)$

41. $g(f(4))$ 42. $f(g(2))$

43. **Critical Thinking** Can you use the tables to find $f(g(4))$? Explain your answer.



44. **Write About It** Is the sum of two linear functions also a linear function? Is the product of two linear functions also a linear function? Explain.

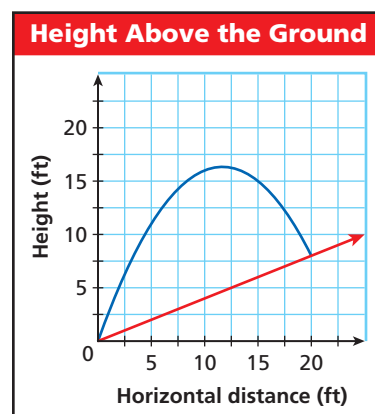
x	2	3	4	5
$f(x)$	0	1	2	3

x	1	2	3	4
$g(x)$	1	2	4	8

45. If $(f \circ g)(x) = (3x + 4)^2$, which of the following could be true?
 (A) $f(x) = 3x + 4$ and $g(x) = x^2$ (C) $f(x) = (3x)^2$ and $g(x) = 4^2$
 (B) $f(x) = x^2$ and $g(x) = 3x + 4$ (D) $f(x) = 3x + 4$ and $g(x) = \sqrt{x}$
46. If $f(x) = 2x + 1$ and $g(x) = 5x - 2$, then which of the following is $(fg)(5)$?
 (F) 253 (H) 47
 (G) 53 (J) 13
47. Given $f(x) = 4 - x^2$ and $g(x) = \frac{1}{2}x - 2$, which of the following is $(f \circ g)(x)$?
 (A) $(f \circ g)(x) = -\frac{1}{2}x^2$ (C) $(f \circ g)(x) = -\frac{1}{2}x^3 + 2x^2 + 2x - 8$
 (B) $(f \circ g)(x) = -\frac{1}{4}x^2 + 2x$ (D) $(f \circ g)(x) = -x^2 + \frac{1}{2}x + 2$
48. **Gridded Response** Given that $f(x) = (x + 1)^2$ and $g(x) = 3x$, find $(f + g)(2)$.

CHALLENGE AND EXTEND

49. Given $f(x) = 2x - 6$ and $f(g(x)) = 3x^2 + 4$, find $g(x)$.
50. Given $f(x) = 3x + 8$ and $g(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 5x + 2 & \text{if } x \geq 0 \end{cases}$, find $g(f(x))$.
51. **Physics** When a ball is thrown up a hill, the height y of the ball is given by the function $y = -0.12x^2 + 2.8x$, where x is the horizontal distance from the thrower. The hill is represented by the linear function $y = \frac{2}{5}x$.
- Find the maximum height of the ball above the ground.
 - Find the height of the ball when it hits the ground.



SPIRAL REVIEW

52. **Business** The value of a computer purchased for \$800 depreciates by 20% each year. (Lesson 7-1)
- Write a function to model the value of the computer after t years.
 - How much will the computer be worth in 10 years?

Decide whether the data set is exponential, and if it is, use exponential regression to find a function that models the data. (Lesson 7-8)

53.

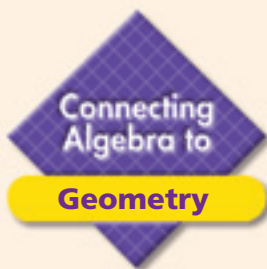
x	2	3	4	5	6
y	5	10	20	40	80

54.

x	1	2	3	4	5
y	5	10	15	20	25

Given $f(x) = \begin{cases} 8x & x \geq 0 \\ x - 9 & x < 0 \end{cases}$, write the rule for each function. (Lesson 9-3)

55. $g(x)$, a horizontal translation of $f(x)$ 5 units to the left
56. $h(x)$, a vertical stretch by a factor of 3



Connecting
Algebra to

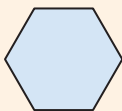
Geometry

See Skills Bank
page S63

Using Geometric Formulas

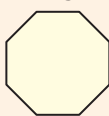
Geometric formulas can be used to find lengths of sides or edges. Solve the formula for the variable that you need. In these formulas, s is the length of a side or an edge and r is the radius.

Regular
Hexagon



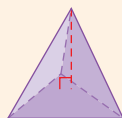
$$A = \frac{3s^2}{2}\sqrt{3}$$

Regular
Octagon



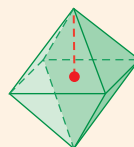
$$A = 2s^2(\sqrt{2} + 1)$$

Regular
Tetrahedron



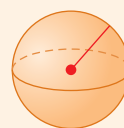
$$V = \frac{s^3}{12}\sqrt{2}$$

Regular
Octahedron



$$V = \frac{s^3}{3}\sqrt{2}$$

Sphere



$$A = 4\pi r^2$$

$$V = \frac{4\pi r^3}{3}$$

Example

A rectangle is 30 cm long and 10 cm wide. Find the length of the sides of a regular octagon that has the same area as this rectangle.

- 1 Find the area of the rectangle.

$$30 \cdot 10 = 300$$

The area is 300 cm^2 .

- 2 Use the octagon formula. Solve for s , the length of one side.

$$A = 2s^2(\sqrt{2} + 1)$$

$$\frac{A}{2(\sqrt{2} + 1)} = s^2$$

Divide both sides by $2(\sqrt{2} + 1)$.

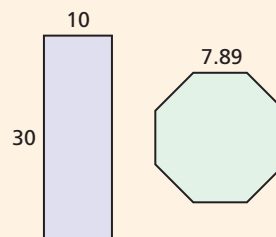
$$s = \sqrt{\frac{A}{2(\sqrt{2} + 1)}}$$

Take the square root of both sides.

- 3 Substitute 300 for the area A , and solve for the side length.

$$s \approx \sqrt{\frac{300}{2(1.41 + 1)}} \approx \sqrt{\frac{300}{4.82}} \approx \sqrt{62.24} \approx 7.89$$

Use 1.41 as an approximation for $\sqrt{2}$.



Try This

Solve each problem. Start by solving the appropriate formula for s or r .

- How large of a sphere could you build with 1000 cubic feet of clay?
- Jake used 540 square inches of mosaic tile to build a tabletop in the shape of a regular hexagon. How long is one edge of this tabletop?
- A tent shaped like a tetrahedron has 30 cubic feet of air space. Describe the base of this tent.
- Cyndi wants to construct an octahedron that has the same volume as a sphere with a radius of 9 cm. What length should she make each edge of the octahedron?



9-5

Functions and Their Inverses

Objectives

Determine whether the inverse of a function is a function.

Write rules for the inverses of functions.

Vocabulary

one-to-one function

Who uses this?

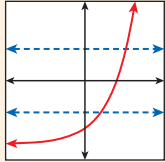
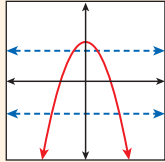
Nurses can use inverse functions to approximate the ages of infants. (See Exercise 37.)

In Lesson 7-2, you learned that the inverse of a function $f(x)$ “undoes” $f(x)$. Its graph is a reflection across the line $y = x$. The inverse may or may not be a function.

Recall that the vertical-line test (Lesson 1-6) can help you determine whether a relation is a function. Similarly, the *horizontal-line test* can help you determine whether the inverse of a function is a function.

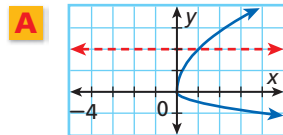


Horizontal-line Test

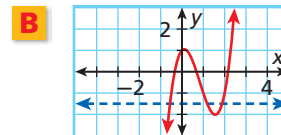
WORDS	EXAMPLES
If any horizontal line passes through more than one point on the graph of a relation, the inverse relation is not a function.	 Inverse is a function.  Inverse is not a function.

EXAMPLE 1 Using the Horizontal-Line Test

Use the horizontal-line test to determine whether the inverse of each relation is a function.



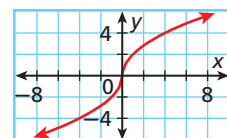
The inverse is a function because no horizontal line passes through two points on the graph.



The inverse is not a function because a horizontal line passes through more than one point on the graph.



1. Use the horizontal-line test to determine whether the inverse of the relation is a function.



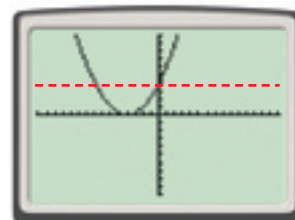
Recall from Lesson 7-2 that to write the rule for the inverse of a function, you can exchange x and y and solve the equation for y . Because the values of x and y are switched, the domain of the function will be the range of its inverse and vice versa.

EXAMPLE 2 Writing Rules for Inverses

Find the inverse of $f(x) = \left(\frac{1}{2}x + 2\right)^2$. Determine whether it is a function, and state its domain and range.

Step 1 Graph the function.

The horizontal-line test shows that the inverse is not a function. Note that the domain of f is all real numbers and the range is $\{y | y \geq 0\}$.



Step 2 Find the inverse.

$$y = \left(\frac{1}{2}x + 2\right)^2$$

Rewrite the function using y instead of $f(x)$.

$$x = \left(\frac{1}{2}y + 2\right)^2$$

Switch x and y in the equation.

$$\sqrt{x} = \sqrt{\left(\frac{1}{2}y + 2\right)^2}$$

Take the square root of both sides.

$$\pm\sqrt{x} = \frac{1}{2}y + 2$$

Note the domain restriction $x \geq 0$.

$$\pm\sqrt{x} - 2 = \frac{1}{2}y$$

Subtract 2 from each side.

$$y = 2(\pm\sqrt{x} - 2)$$

Isolate y .

$$y = \pm 2\sqrt{x} - 4$$

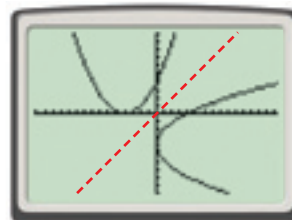
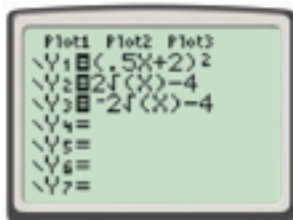
Simplify.

Because of the $\pm$ symbol, there may be two y -values for an x -value. This confirms that the inverse is not a function. Because the inverse is not a function, you cannot use the notation $f^{-1}(x)$.

The domain of the inverse is the range of $f(x)$: $\{x | x \geq 0\}$. The range is the domain of $f(x)$: all real numbers.

Check Graph both relations to see that they are symmetric about $y = x$.

To graph the inverse, you will have to graph the positive and negative cases separately.



Caution!

When you take the square root of both sides of a quadratic equation be sure to add the $\pm$ symbol. (Lesson 5-4)



2. Find the inverse of $f(x) = x^3 - 2$. Determine whether it is a function, and state its domain and range.

You have seen that the inverses of functions are not necessarily functions. When both a relation and its inverse are functions, the relation is called a *one-to-one function*. In a **one-to-one function**, each y -value is paired with exactly one x -value.

You can use composition of functions to verify that two functions are inverses. Because inverse functions “undo” each other, when you compose two inverses the result is the input value x .

Identifying Inverse Functions

WORDS	ALGEBRA	EXAMPLE
If the compositions of two functions equal the input value, the functions are inverses.	If $f(g(x)) = g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverse functions.	$f(x) = 3x$ and $g(x) = \frac{1}{3}x$ $f(g(x)) = 3\left(\frac{1}{3}x\right) = x$ $g(f(x)) = \frac{1}{3}(3x) = x$

EXAMPLE 3 Determining Whether Functions Are Inverses

Determine by composition whether each pair of functions are inverses.

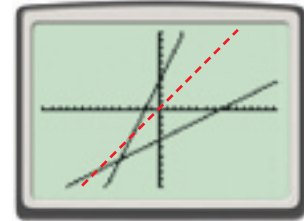
A $f(x) = 2x + 4$ and $g(x) = \frac{1}{2}x - 4$

Find the composition $f(g(x))$.

$$\begin{aligned}
 f(g(x)) &= 2\left(\frac{1}{2}x - 4\right) + 4 && \text{Substitute } \frac{1}{2}x - 4 \text{ for } x \text{ in } f. \\
 &= (x - 8) + 4 && \text{Use the Distributive Property.} \\
 &= x - 4 && \text{Simplify.}
 \end{aligned}$$

Because $f(g(x)) \neq x$, f and g are not inverses. There is no need to check $g(f(x))$.

Check The graphs are not symmetric about the line $y = x$.



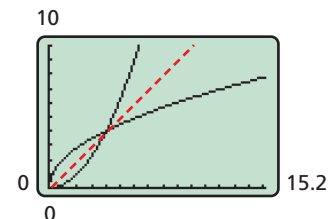
B For $x \geq 0$, $f(x) = \frac{1}{4}x^2$ and $g(x) = 2\sqrt{x}$.

Find the compositions $f(g(x))$ and $g(f(x))$.

$$\begin{aligned}
 f(g(x)) &= \frac{1}{4}(2\sqrt{x})^2 && g(f(x)) = 2\sqrt{\frac{1}{4}x^2} \\
 &= \frac{1}{4}(4x) && = 2\left(\frac{1}{2}x\right) \\
 &= x, x \geq 0 && = x
 \end{aligned}$$

Because $f(g(x)) = g(f(x)) = x$ for $x \geq 0$, f and g are inverses.

Check The graphs are symmetric about the line $y = x$ when $x \geq 0$.



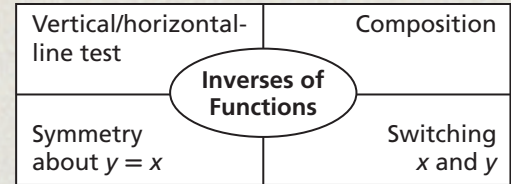
Determine by composition whether each pair of functions are inverses.

3a. $f(x) = \frac{2}{3}x + 6$ and $g(x) = \frac{3}{2}x - 9$

3b. $f(x) = x^2 + 5$ and $g(x) = \sqrt{x} - 5$ for $x \geq 0$

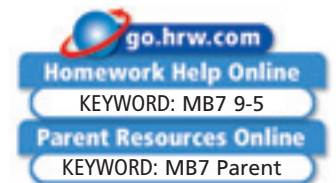
THINK AND DISCUSS

1. Explain why the horizontal-line test works.
2. Explain the relationship between the domain and range of a function and the domain and range of its inverse.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Describe how each method or characteristic is used to find or verify inverses.



9-5

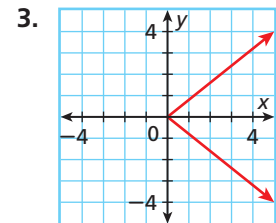
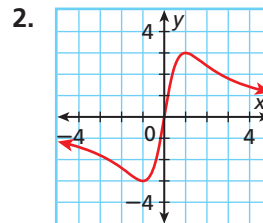
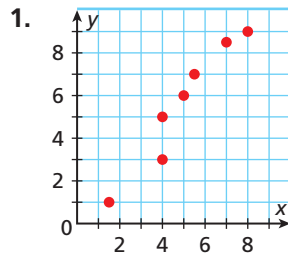
Exercises



GUIDED PRACTICE

SEE EXAMPLE 1
p. 690

Use the horizontal-line test to determine whether the inverse of each relation is a function.



SEE EXAMPLE 2
p. 691

Find the inverse of each function. Determine whether the inverse is a function, and state its domain and range.

4. $f(x) = -3x + 21$

5. $g(x) = x^2 - 9$

6. $h(x) = \frac{x+5}{8}$

SEE EXAMPLE 3
p. 692

Determine by composition whether each pair of functions are inverses.

7. $f(x) = 4x - 12$ and $g(x) = -4x + 8$

8. $f(x) = \sqrt{3x}$ and $g(x) = \frac{x^2}{3}$ for $x \geq 0$

PRACTICE AND PROBLEM SOLVING

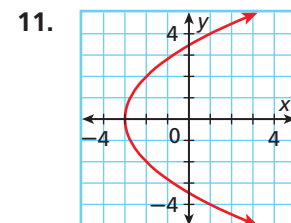
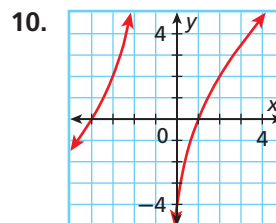
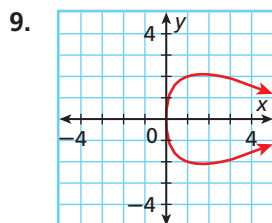
Independent Practice

For Exercises	See Example
9–11	1
12–17	2
18–21	3

Extra Practice

Skills Practice p. S21
Application Practice p. S40

Use the horizontal-line test to determine whether the inverse of each relation is a function.



Find the inverse of each function. Determine whether the inverse is a function, and state its domain and range.

12. $f(x) = \frac{3}{5}x$

13. $f(x) = 8x^3$

14. $f(x) = \frac{x}{x+1}$

15. $f(x) = \frac{5x+9}{6}$

16. $f(x) = (x-4)^2$

17. $f(x) = 5 + \sqrt{x+8}$

Determine by composition whether each pair of functions are inverses.

18. $f(x) = \frac{5-2x}{9}$ and $g(x) = -\frac{9}{2}x + \frac{5}{2}$

19. $f(x) = \frac{5}{x+1}$ and $g(x) = \frac{x-1}{5}$ for $x \neq -1$

20. $f(x) = 3\sqrt{x}$ and $g(x) = \frac{1}{3}x^2$ for $x \geq 0$

21. $f(x) = \log \frac{x}{2}$ and $g(x) = 2(10^x)$ for $x > 0$

22. Biology The number of times that a cricket chirps per minute can be found by using the function $N(F) = 4F - 160$, where F is the temperature in degrees Fahrenheit.

- Find and interpret the inverse of $N(F)$.
- What is the temperature when the cricket is chirping 60 times a minute?
- How many times will the cricket chirp in 1 minute at a temperature of 80°F ?

23. Business The managers of a pizza restaurant have found that the function $t(d) = 20 + 2.5d$ models the length of time in minutes, after it is ordered, for a pizza to be delivered a distance of d miles.

- Write the inverse of $t(d)$, and explain what it represents.
- How far away can a customer live and still get a pizza within 30 minutes of placing an order?

Write the rule for the inverse of each function. Then state its domain and range.

24. $f(x) = \frac{7-8x}{3}$

25. $f(x) = \frac{5}{x+4}$

26. $f(x) = 5(x+6)^2$

27. $f(x) = \sqrt[3]{x-12}$

28. $f(x) = \frac{x^3-5}{12}$

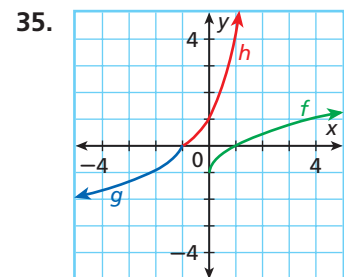
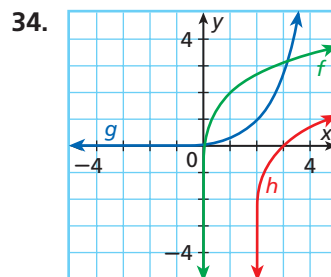
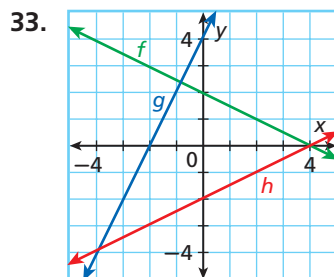
29. $f(x) = 7^x$

30. $f(x) = \ln(x+2)$

31. $f(x) = 3e^{x+5}$

32. $f(x) = \frac{\log(x+8)}{4}$

For each graph, determine which two functions are inverses.



36. Statistics A person's standardized score on a test is given by the function

$$z(x) = \frac{x-250}{40}, \text{ where } x \text{ is the actual score on the test.}$$

- Find and interpret the inverse of $z(x)$.
- If a person's standardized score on a test was 2.5, what was the person's actual score on the test?

37. Medicine Nurses carefully track the height and weight of infants to ensure that they are healthy as they grow. The average height in inches of a girl in the first 3 years of life can be modeled by $h(a) = 3\sqrt{a} + 19$, where a is the age of the girl in months.

- Find and interpret the inverse of $h(a)$.
- Estimation** Estimate the age of a girl whose height is $32\frac{1}{2}$ inches.



Biology



There are about 900 species of crickets worldwide. Many male crickets make chirping noises either by rubbing their wings together or by rubbing their wing cases against their hind legs.



38. This problem will prepare you for the Multi-Step Test Prep on page 706.

If an object is dropped from a hot-air balloon at an altitude of 500 ft, the object's height after t seconds can be modeled by $h(t) = -16t^2 + 500$.

- Find and interpret the inverse of $h(t)$.
- How long does it take the object to hit the ground?
- How long does it take the object to fall if it lands on the roof of a building that is 128 ft tall?

39. **Conservation** As a brown bear with a radio collar walks along a river, the distance from the bear to an observation post after t seconds is given by the function $d(t) = \sqrt{1600 + 9t^2}$.

- Find and interpret the inverse of $d(t)$.
- If the tracking equipment has a range of 5500 feet, how long will a person in the observation post be able to track the bear before having to move?

40. **Photography** The cost in dollars of enlarging a photo is given by the function $c = 0.1\ell^2$, where ℓ is the length of the enlargement in inches.

- Write the inverse of the function, and explain what it represents.
- Determine the length of an enlargement that costs \$25.60.

41. **Manufacturing** The surface area of an aluminum can is given by the function $S(h) = 18\pi + 6\pi h$.

- Find and interpret the inverse of $S(h)$.
- Find the height to the nearest hundredth of a centimeter of a can with a surface area of 500 cm^2 .



42. **Critical Thinking** Identify two functions that are their own inverses.



43. **Geometry** The area of a square is $A = s^2$, where s is the length of a side.

- Find and interpret the inverse of the function.
- Explain how an architect or city planner might use the inverse.
- Estimation** Estimate the side length of a square park that covers an area of 800,000 square feet.

44. **Critical Thinking** If a relation is not a function, can its inverse be a function? Use an example to illustrate your answer.



45. **Write About It** Describe two ways to determine whether two functions are inverses of each other. How would your methods apply to determining whether a function is its own inverse?

46. The formula for converting from degrees Celsius to degrees Fahrenheit is $F = \frac{9}{5}C + 32$. Which of the following formulas converts degrees Fahrenheit to degrees Celsius?

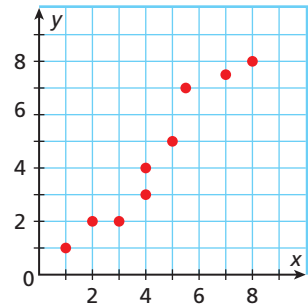
(A) $C = \frac{9}{5}F - 32$

(C) $C = \frac{5}{9}F + 32$

(B) $C = \frac{9}{5}(F - 32)$

(D) $C = \frac{5}{9}(F - 32)$

47. Which of the following is true about the relation graphed?
- (F) Both the relation and its inverse are functions.
 - (G) The relation is a function, but its inverse is not a function.
 - (H) The relation is not a function, but its inverse is a function.
 - (J) Neither the relation nor its inverse is a function.



48. Which of the following is the inverse of $f(x) = \sqrt{x} + 1$?
- (A) $f^{-1}(x) = x^2 - 1, x \geq 0$
 - (B) $f^{-1}(x) = (x - 1)^2, x \geq 0$
 - (C) $f^{-1}(x) = x^2 + 1, x \geq 0$
 - (D) $f^{-1}(x) = (x + 1)^2, x \geq 0$
49. For a certain function, $f(0) = 2$ and $f^{-1}(4) = 1$. Which of the following is also true?
- (F) $f^{-1}(0) = 2$
 - (G) $f^{-1}(2) = 0$
 - (H) $f(4) = 1$
 - (J) $f(2) = 0$
50. Which function has an inverse that is NOT a function?
- (A) $f(x) = 2x^3 + 3$
 - (B) $f(x) = \sqrt{2x} + 5$
 - (C) $f(x) = x^2 - 1$
 - (D) $f(x) = 3^x + 1$
51. **Short Response** Given that $f(x)$ is a quadratic function, is its inverse a function? Explain.

CHALLENGE AND EXTEND

52. Use the Quadratic Formula to find a rule for the inverse of $f(x) = 3x^2 - 6x - 9$.

Find a rule for the inverse of each function.

53. $f(x) = \frac{3 + \ln x}{3 - \ln x}$

54. $f(x) = \frac{5 \log(x^3) - 3 \log(x^2)}{3}$

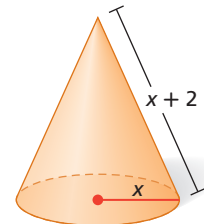
55. Is $f^{-1}(g^{-1}(x)) = (f(g(x)))^{-1}$ correct? Support your answer.

56. **Personal Finance** A financial manager predicts that if a person leaves \$1000 in his mutual fund, the value of the money after t years will be $V(t) = 1000(1.08^t)$.
- a. Find and interpret the inverse of $V(t)$.
 - b. If a person puts \$1000 into this mutual fund, predict how much money the person will have in the fund in 10 years.
 - c. If a person puts \$1000 into this mutual fund, after how many years will the person have \$4000 in the fund?

SPIRAL REVIEW



57. **Geometry** Find a polynomial expression for the surface area of the cone that is shown (*Hint: $S = B + \pi r \ell$*) (Lesson 6-1)



Find each product. (Lesson 6-2)

58. $(x + 2)(x^4 + x^2 + 1)$

59. $(x + 3)(x^2 - 3x + 9)$

60. $(x^3 - x^2)(x^2 + 2x)$

Given $f(x) = 9x - 5$ and $g(x) = x^2 - x - 1$, find each function. (Lesson 9-4)

61. $(f + g)(x)$

62. $(f - g)(x)$

63. $(g - f)(x)$



Explore Differences and Ratios

Recall that common differences or common ratios can help you determine which model is appropriate for a data set. You can use spreadsheets to quickly evaluate differences and ratios in data sets.

Use with Lesson 9-6

Activity

Consider the data set shown in the table. Use a spreadsheet to find the first differences, second differences, and ratios of the y -values. Identify which, if any, are constant.

x	5	6	7	8	9	10	11
y	1	3.4	6.6	10.6	15.4	21	27.4

Enter the row heads and the data into the spreadsheet as shown. Remember that the x -values must be evenly spaced.

- Find first differences. In cell C3, enter the formula $=C2-B2$ to find the difference of the first two y -values. Copy this formula into cells D3 through H3.

	A	B	C	D	E	F	G	H
1	x	5	6	7	8	9	10	11
2	y	1	3.4	6.6	10.6	15.4	21	27.4
3	1st differences		2.4	3.2	4	4.8	5.6	6.4
4	2nd differences							
5	Ratios							

- Find second differences. In cell D4, enter the formula $=D3-C3$ to find the first second difference. Copy this formula into cells E4 through H4.

	A	B	C	D	E	F	G	H
1	x	5	6	7	8	9	10	11
2	y	1	3.4	6.6	10.6	15.4	21	27.4
3	1st differences		2.4	3.2	4	4.8	5.6	6.4
4	2nd differences			0.8	0.8	0.8	0.8	0.8
5	Ratios							

- Find the ratios. In cell C5, enter the formula $=C2/B2$ to find the ratio of the first two y -values. Copy this formula into cells D5 through H5.

	A	B	C	D	E	F	G	H
1	x	5	6	7	8	9	10	11
2	y	1	3.4	6.6	10.6	15.4	21	27.4
3	1st differences		2.4	3.2	4	4.8	5.6	6.4
4	2nd differences			0.8	0.8	0.8	0.8	0.8
5	Ratios		3.4	1.94	1.61	1.45	1.36	1.3

For this data set, the second differences are constant. You can adjust the spreadsheet to include more data points if necessary.

Try This

For each data set, find the first differences, second differences, and ratios of the y -values. Identify which, if any, are constant.

- | | | | | | |
|-----|-----|-----|-----|-----|------|
| x | 2 | 4 | 6 | 8 | 10 |
| y | -20 | 124 | 364 | 700 | 1132 |

- | | | | | | | |
|-----|------|------|------|------|------|------|
| x | 4 | 7 | 10 | 13 | 16 | 19 |
| y | 1.31 | 2.48 | 3.65 | 4.82 | 5.99 | 7.16 |

- | | | | | | |
|-----|------|-------|--------|--------|--------|
| x | 3 | 4 | 5 | 6 | 7 |
| y | 8.96 | 35.84 | 143.36 | 573.44 | 2293.8 |

- | | | | | | |
|-----|-----|----|-----|-----|-----|
| x | 1.5 | 3 | 4.5 | 6 | 7.5 |
| y | -9 | 15 | 57 | 117 | 195 |

- Critical Thinking** What type of function might be an appropriate model for Problem 4? Justify your response.



9-6

Modeling Real-World Data



Objectives

Apply functions to problem situations.

Use mathematical models to make predictions.

Who uses this?

You can use mathematical models to analyze and predict the number of automated teller machines (ATMs) in use. (See Example 3.)

Much of the data that you encounter in the real world may form a pattern. Many times the pattern of the data can be modeled by one of the functions you have studied. You can then use the functions to analyze trends and make predictions. Recall some of the parent functions that you have studied so far.



Helpful Hint

Because the square-root function is the inverse of the quadratic function, the constant differences for x - and y -values are switched.

Families of Functions				
Family	Linear	Quadratic	Exponential	Square Root
Rule	$f(x) = x$	$f(x) = x^2$	$f(x) = b^x, b > 0$	$f(x) = \sqrt{x}$
Graph				
Constant Differences or Ratios	Constant first differences between y -values for evenly spaced x -values	Constant second differences between y -values for evenly spaced x -values	Constant ratios between y -values for evenly spaced x -values	Constant second differences between x -values for evenly spaced y -values

EXAMPLE 1

Identifying Models by Using Constant Differences or Ratios

Use constant differences or ratios to determine which parent function would best model the given data set.

A The length of a spring depends on the mass attached.

Mass (kg)	4	5	6	7	8	9	10
Length (cm)	30.6	32	33.4	34.8	36.2	37.6	39

Notice that the mass data are evenly spaced. Check the first differences between the lengths to see if the data set is linear.

Length (cm)	30.6	32	33.4	34.8	36.2	37.6	39
-------------	------	----	------	------	------	------	----

First differences 1.4 1.4 1.4 1.4 1.4 1.4

Because the first differences are a constant 1.4, a linear model will best model the data.

Use constant differences or ratios to determine which parent function would best model the given data set.

B The age of a tree can be determined from its diameter.

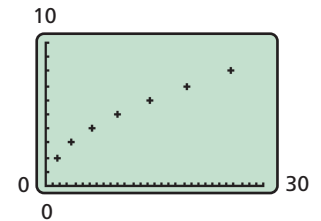
Diameter (cm)	1.6	3.6	6.4	10.0	14.4	19.6	25.6
Age (yr)	2	3	4	5	6	7	8

Notice that the age data are evenly spaced. Check the first differences between diameters.

Diameter (cm)	1.6	3.6	6.4	10.0	14.4	19.6	25.6
First differences	2	2.8	3.6	4.4	5.2	6	
Second differences	0.8	0.8	0.8	0.8	0.8		

Because the second differences of the independent variable are constant when the dependent variables are evenly spaced, a square-root function will best model the data.

Check A scatter plot reveals a shape similar to the square-root parent function $f(x) = \sqrt{x}$.



C The volume of a liquid remaining after evaporation depends on the time elapsed.

Time (h)	1	2	3	4	5	6
Volume (mL)	512	384	288	216	162	121.5

Because the time data are evenly spaced, check the differences between the volumes.

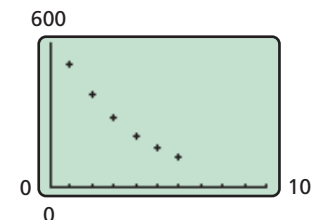
Volume (mL)	512	384	288	216	162	121.5
First differences	-128	-96	-72	-54	-40.5	
Second differences	32	24	18	13.5		

Neither the first nor second differences are constant. Check ratios between the volumes.

$$\frac{384}{512} = 0.75, \frac{288}{384} = 0.75, \frac{216}{288} = 0.75, \frac{162}{216} = 0.75, \text{ and } \frac{121.5}{162} = 0.75.$$

Because the ratios between the values of the dependent variable are constant, an exponential function would best model the data.

Check A scatter plot reveals a shape similar to an exponential decay function.



Use constant differences or ratios to determine which parent function would best model the given data set.

1a.

x	12	48	108	192	300
y	10	20	30	40	50

1b.

x	21	22	23	24
y	243	324	432	576

Real-world data rarely have differences or ratios that are mathematically constant, but you can analyze them to see if they are close to constant. You can also use a scatter plot to visually determine which model best suits a data set. Then you can perform a regression to find a function to model the data. Recall that the correlation coefficient r helps you see how well the model fits the data (Lesson 2-7).

EXAMPLE 2 Conservation Application

A zoologist is monitoring the size of a herd of buffalo in the years since the herd was released into a wilderness area. Write a function that models the given data.

Time (yr)	5	6	7	8	9	10
Buffalo	124	150	185	213	261	322



Step 1 Make a scatter plot of the data.

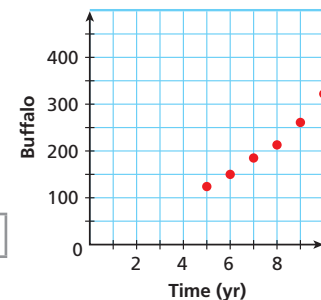
The data appear to form a quadratic or an exponential pattern.

Step 2 Analyze differences.

Buffalo	124	150	185	213	261	322
---------	-----	-----	-----	-----	-----	-----

First differences 26 35 28 48 61

Second differences 9 -7 20 13

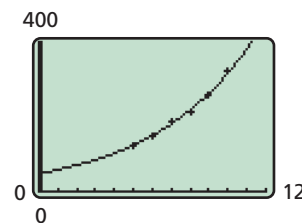
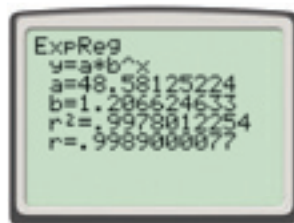


Step 3 Neither the first nor the second differences are close to constant, so analyze the ratios.

$$\frac{150}{124} = 1.210, \frac{185}{150} = 1.233, \frac{213}{185} = 1.151, \frac{261}{213} = 1.225, \text{ and } \frac{322}{261} = 1.234$$

The ratios are all close to 1.2, indicating that an exponential model would be appropriate.

Step 4 Use your graphing calculator to perform an exponential regression.



An exponential function that models the data is $f(x) = 48.581(1.207^x)$. The correlation coefficient r is very close to 1, which indicates a good fit.

Helpful Hint

To display the correlation coefficient r on some calculators, you must turn on the diagnostic mode.

Press **2nd** **CATALOG** **0**, and choose **DiagnosticOn**.



2. Write a function that models the given data.

x	12	14	16	18	20	22	24
y	110	141	176	215	258	305	356

When data are not ordered or evenly spaced, you may have to try several models to determine which best approximates the data. Graphing calculators often indicate the value of the *coefficient of determination*, indicated by r^2 or R^2 . The closer the coefficient is to 1, the better the model approximates the data.

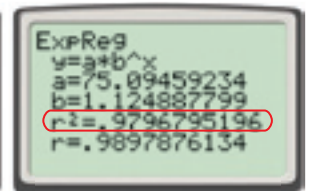
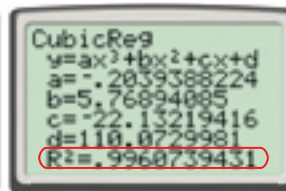
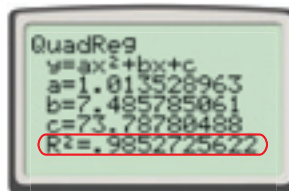
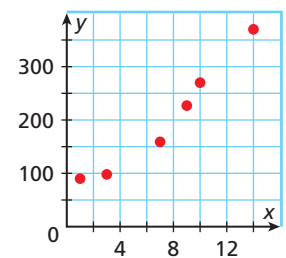
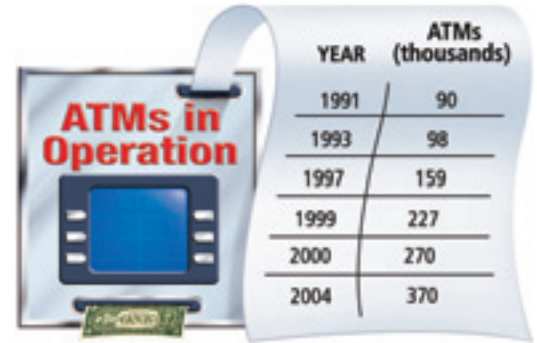
EXAMPLE 3 Banking Application

The data set shows the approximate number of automated teller machines (ATMs) in operation in the United States. Using 1990 as a reference year, write a function that models the data.

The data are not evenly spaced, so you cannot analyze differences or ratios.

Create a scatter plot of the data. Use 1990 as year 0. The data appears to be quadratic, cubic, or exponential.

Use the calculator to perform each type of regression.



Compare the values of r^2 . The cubic model seems to be the best fit. The function $f(x) \approx 0.2x^3 + 5.44x^2 - 22.13x + 110.07$ models the data.

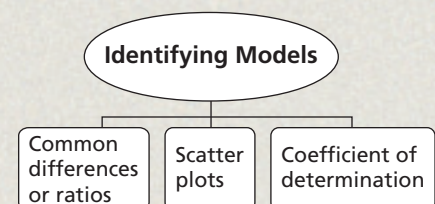


3. Write a function that models the data.

Fertilizer/Acre (lb)	11	14	25	31	40	50
Yield/Acre (bushels)	245	302	480	557	645	705

THINK AND DISCUSS

1. Explain the limitations of finding constant differences or ratios when working with real-world data.
2. **GET ORGANIZED** Copy and complete the graphic organizer. Explain how each method can help you determine which model best fits a data set.





GUIDED PRACTICE

SEE EXAMPLE 1

p. 698

1 Use constant differences or ratios to determine which parent function would best model the given data set.

1.

x	y
6	69.6
13	51.4
20	33.2
27	15
34	-3.2
41	-21.4

2.

x	y
11	2
47	6
99	10
167	14
251	18
351	22

3.

x	y
0	125
1	150
2	180
3	216
4	259.2
5	311.04

SEE EXAMPLE 2

p. 700

4. This table shows the mass in grams m of the radioactive substance iodine-131 remaining in a container t days after the beginning of an experiment.

Time t (days)	0	1	2	3	4	5	6
Mass m (g)	1000	917.40	841.62	772.10	708.33	649.82	596.14

- Write a function that models the data.
- Use your model to predict the number of grams of iodine-131 that will be left after 20 days.
- Use your model to predict when there will be less than 50 grams remaining.

SEE EXAMPLE 3

p. 701

5. The table shows the value of a stock at various points in the past 24 months since Carla bought the stock.

Time t (months)	0	4	9	12	15	20	24
Stock Value v (\$)	62	54	45	48	55	53	60

- Write a function that models the data.
- Use your model to predict the stock price 6 months after Carla bought it.
- Would you recommend that Carla use your model to predict the value of her stock a year from now? Why or why not?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
6–8	1
9	2
10	3

Extra Practice

Skills Practice p. S21

Application Practice p. S40

Use constant differences or ratios to determine which parent function would best model the given data set.

6.

x	y
1	380
3	343
5	310
7	279
9	252
11	228

7.

x	y
2	97
8	202
14	253
20	250
26	193
32	82

8.

x	y
4	4
9	6
16	8
25	10
36	12

9. **Agriculture** A farmer is experimenting with the amount of fertilizer to put on his corn fields. Different amounts of fertilizer are applied to each field, and the resulting yields are measured. Write a function that models the given data.

Fertilizer/Acre (lb)	45	70	90	115	125	135	150
Yield/Acre (bushels)	29	60	70	88	84	86	76

$$f(x) \approx -0.009x^2 + 2.28x - 55.31$$

10. **Biology** The table shows the estimated number of *E. coli* bacteria in a lab dish t minutes after the start of an experiment.

Time (min)	0	10	20	30	40	50	60
Bacteria	300	423	596	842	1188	1686	2354

- Using t as the independent variable, find the model that best fits the data. $n(t) \approx 299.92(1.035^t)$
- Use your model to predict the number of bacteria after 3 hours. **about 146,940**
- How long does it take the population of *E. coli* to triple? **about 32min**



11. **Real Estate** The table shows the prices of some recent home sales compared with the area of the homes.

$$y \approx 34.37x + 85,851.76$$

- Using area as the independent variable, find a model for the data. **about 2594 ft²**
- Use your model to predict the number of square feet in a house that is priced at \$175,000.
- How accurate do you think your answer to part b is?

Area of Homes Sold	
Area (ft ²)	Price (\$)
2675	179,000
1170	125,900
1486	136,750
2510	172,500
2444	169,900
2980	187,000

12. **Economics** An economist is studying the median yearly income of workers by their ages.

Age (yr)	18	28	38	48	58	68
Median Income (\$)	17,480	30,650	37,440	41,230	37,570	21,390

- Find an appropriate model for the data.
- Use your model to predict the median income for a worker who is 43 years old. **about \$40,780**



13. **Data Collection** Use a graphing calculator and a motion detector to measure the distance of a ball or a toy car as it travels down a ramp. Set the motion detector at the top of the ramp and release the object to collect the data.

- Find an appropriate model for distance versus time. **Answers will vary.**
- Use your model to predict the distance the object would travel in 1 minute if the ramp continued indefinitely. **Answers will vary.**

14. **Health** The table shows the mean age of mothers in the United States when they had their first child.

Year	1980	1985	1990	1995	2000
Mean Age of Mother at First Birth	22.7	23.7	24.2	24.5	24.9

- Using 1980 as a reference year, find both a quadratic and cubic model for the data.
- Use both models to predict the mean age of a mother at first birth in 2010.
- Explain which prediction you think is more accurate.



Math History



In 1806, French mathematician Adrien-Marie Legendre (1752–1833) modeled the orbit of a comet by fitting a quadratic curve to location data.



15. This problem will prepare you for the Multi-Step Test Prep on page 706.

The table shows how the volume v of air in a hot-air balloon relates to the temperature t of the air.

- Find an exponential model for the data.
- Use your model to predict the volume of the air when its temperature is 109°F .
- Would your model be accurate for any air temperature greater than 118°F ? Why or why not?

Temperature ($^{\circ}\text{F}$)	Volume (ft^3)
100	30,000
106	33,000
112	36,300
118	39,930

16. **Agriculture** The table shows the number and the average area of farms in the United States in the last century.

- Using the number of farms as the independent variable, find a model for the average size of farms.
- Use your model to predict the average size of the farms when the number of farms reaches 1 million.
- Use your model to estimate the average size of the farms when there were 4.5 million farms.

Number and Size of U.S. Farms		
Year	Farms (millions)	Average Area (acres)
1910	6.4	139
1930	6.3	157
1950	5.4	216
1969	2.7	390
1987	2.1	462
1997	1.9	487

17. **Baseball** The Fan Cost Index tracks the cost for a family of four to attend a Major League Baseball game.

Year	1991	1994	1997	2000	2003
FCI	\$79.41	\$96.41	\$107.26	\$132.44	\$151.19



- Find a model for the data. Use 1990 as year 0.
- How fast has the FCI been increasing according to your model?
- The FCI in 2004 was \$155.52. How close is the actual value to the value predicted by your model?
- Use your model to predict when the FCI will reach \$200.
- Because of inflation, something that cost \$1.00 in 1991 cost \$1.34 in 2003. How does the change in the FCI compare with inflation?

18. **Biology** The table shows the number of species of reptiles and amphibians and the area in square miles for some islands in the Caribbean.

- Using number of species as the independent variable, find an appropriate model for the data.
- Use your model to predict the area of an island with 75 species of reptiles and amphibians.
- How accurate do you think your prediction in part **b** is? Explain.

Species	Area (mi^2)
11	5
16	32
53	3,435
45	4,244
108	29,371
100	44,218



Green iguana

19. **Critical Thinking** Sometimes data that appear linear are better modeled by a quadratic function. What can you conclude about the value of a in the quadratic model of such a data set?



20. **Write About It** Suppose that a model can be found that provides a good fit for data on two variables. What evidence does the model give of a cause-and-effect relationship between the two variables? Use examples in your explanation.

21. Which of the following is true for the data in the table?

- (A) The first differences of values of the dependent variable are constant.
- (B) The second differences of values of the dependent variable are constant.
- (C) The ratios of values of the dependent variable are constant.
- (D) The ratios of values of the independent variable are constant.

x	y
3	2
4	23
5	50
6	83
7	122
8	167

22. Find n so that an exponential model will fit the data exactly.

- (F) $n = 40$
- (G) $n = 49$
- (H) $n = 45$
- (J) $n = 52$

x	5	6	7
y	16	28	n

23. Find n so that a quadratic model will fit the data exactly.

- (A) $n = 60$
- (B) $n = 70$
- (C) $n = 80$
- (D) $n = 90$

x	5	6	7	8
y	12	32	58	n

CHALLENGE AND EXTEND

24. The function $P(t) = \frac{a}{1 + be^{-kt}}$, called a *logistic function*, is often used when there are factors such as food or space that limit a population's growth. The number of fish in a stocked pond can be modeled by the function $F(t) = \frac{4000}{1 + 5.7e^{-0.2t}}$, where t is the number of months after the pond is stocked.

- a. Predict the number of fish in the lake after 10 months.
- b. When will the population of fish reach 3000?
- c. Find the maximum number of fish that the pond can hold if the function is correct.



25. **Graphing Calculator** Another type of regression you can perform is a power regression. Use the **PwrReg** feature to find a model for the given data. Which parent function best fits the data?

x	1	24	41	74
y	1	4.9	6.4	8.6

SPIRAL REVIEW

Graph each system of inequalities. (Lesson 3-3)

26.
$$\begin{cases} y \geq 3x + 1 \\ y \leq x - 3 \end{cases}$$

27.
$$\begin{cases} y \geq x - 8 \\ y \leq -\frac{4}{3}x + \frac{1}{3} \end{cases}$$

28.
$$\begin{cases} y \leq 5x \\ y \geq x + 2 \end{cases}$$

29. **Business** The profit for a company in thousands of dollars is modeled by the function $p(x) = -x^3 + 12x^2 - 12x - 80$, where x is the number of items produced in thousands. (Lesson 5-5)

- a. Find the zeros of the function.
- b. Which zero represents the number of items that the company must produce to break even?

Determine by composition whether each pair of functions are inverses. (Lesson 9-5)

30. $f(x) = x^2 + 1$ and $g(x) = \sqrt{x} + 1$

31. $f(x) = -4 + 5x$ and $g(x) = \frac{1}{5}x + \frac{4}{5}$

MULTI-STEP TEST PREP



Functional Relationships

Full of Hot Air When you ride in a hot-air balloon that is rising vertically, the distance to the farthest object that you can see increases as the balloon's height increases. The table shows data that were collected on a hot-air balloon.

Balloon's Height (m)	7.8	31.3	70.5	125.4	196.0
Distance That You Can See (km)	10	20	30	40	50

1. Which is the independent variable? Why?
2. Which parent function best models the data set? Why?
3. Write a function $f(x)$ that models the data.
4. If you are in a hot-air balloon at a height of 100 m, how far would you expect to be able to see?
5. Write the inverse function of $f(x)$, and explain what it represents.
6. Find the approximate minimum height for a hot-air balloon if you want to be able to see objects that are 25 km away.
7. The function $g(x) = 0.3x$ converts distances in feet to approximate distances in meters. Write a composite function for the distance that you can see in kilometers from a height of x feet.



Quiz for Lessons 9-4 Through 9-6



9-4 Operations with Functions

Given $f(x) = \frac{5}{x+3}$, $g(x) = x - 6$, and $h(x) = x^2 - 4x - 12$, find each function or value.

1. $(f - g)(2)$

2. $(g + h)(x)$

3. $\left(\frac{g}{h}\right)(8)$

4. $\left(\frac{h}{g}\right)(x)$

5. $(gh)(5)$

6. $(gf)(x)$

7. $g(f(-2))$

8. $h(g(x))$

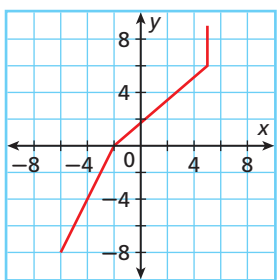
 9. Find $(f \circ g)(x)$. State the domain of the composite function.

 10. Erin receives a 30% employee discount at the camera store where she works. During a sale, she receives an additional 20% off the discounted price. Write a composite function for the price Erin pays for an item with an original price of p dollars.

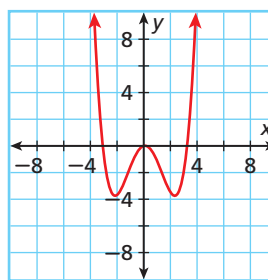

9-5 Functions and Their Inverses

State whether the inverse of each relation is a function.

11.



12.



Write the rule for the inverse of each function. Then state the domain and range of the inverse.

13. $f(x) = \frac{2}{3}x - 12$

14. $g(x) = \frac{12}{x-5}$

15. $h(x) = x^2 - 4$

16. $n(x) = 3^x$



9-6 Modeling Real-World Data

17. Use finite differences or ratios to determine which parent function would best model this set of data.

x	0	1	2	3	4
y	625	375	225	135	81

18. The table shows the average temperature in degrees Fahrenheit and the average utility bill for the households in a town in recent months. Using average temperature as the independent variable, find a model for the average bill.

Average Monthly Temperature ($^{\circ}\text{F}$)	Average Monthly Utility Bill (\$)
61	108
80	103
46	148
72	89
50	125
88	132

Study Guide: Review

Vocabulary

composition of functions . . . 683

one-to-one function 691

piecewise function 662

step function 663

Complete the sentences below with vocabulary words from the list above.

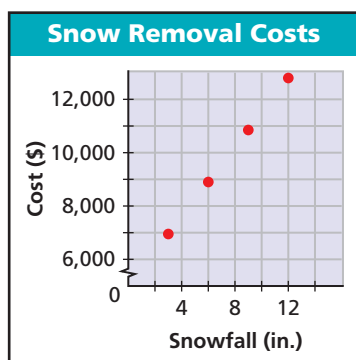
1. In a(n) ? , each y -value is paired with exactly one x -value.
2. A(n) ? is a piecewise function that is constant for each interval of its domain.
3. The function operation that uses the output from one function as the input for a second function is the ? .

9-1 Multiple Representations of Functions (pp. 654–661)**EXAMPLE**

- The managers of a town are interested in the cost of snow removal over the winter. The table shows the cost of removing various amounts of snow. Use a graph and an equation to find the cost to remove 24 inches of snow.

Snowfall (in.)	Cost (\$)
3	6,950
6	8,900
9	10,850
12	12,800

A scatter plot shows that the data is linear.



Find the slope of the line by using two points.

$$m = \frac{8900 - 6950}{6 - 3} = \frac{1950}{3} = 650$$

Write an equation by using one of the points.

$$y - 6950 = 650(x - 3)$$

$$y = 650x + 5000$$

The cost for removing 24 inches of snow is

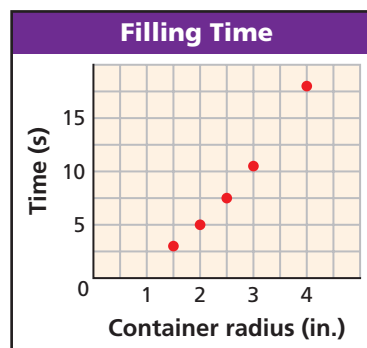
$$y = 5000 + 650(24) = \$20,600.$$

EXERCISES

4. Draw a graph of speed versus time that represents the following situation.

Avery drove 5 miles to her mother's house and visited with her mother for 20 minutes. Then she drove on the freeway for 15 minutes before arriving home.

5. A caterer is planning for a large fund-raising dinner. He plans to have 4 trays of 30 appetizers each on the buffet. In addition, he will prepare an additional 4 appetizers per guest. Create a table, a graph, and an equation to represent the number of appetizers with relation to the number of guests.
6. The scatter plot shows how long it takes to fill various cylindrical containers of different radii.



- a. Create a table and an equation for the data.
- b. Use your equation to predict the time that it would take to fill a cylindrical container with a radius of 7 inches.

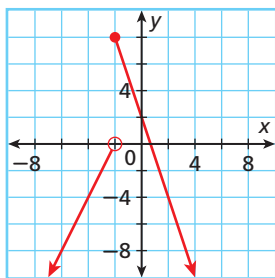
9-2 Piecewise Functions (pp. 662–669)

EXAMPLES

- Evaluate $f(x) = \begin{cases} 5x + 2 & \text{if } x \leq 1 \\ x^2 - 6 & \text{if } x > 1 \end{cases}$ for $x = -2$ and $x = 5$.
- $f(-2) = 5(-2) + 2 = -8$ Use the rule for $x \leq 1$.
- $f(5) = 5^2 - 6 = 19$ Use the rule for $x > 1$.
- Graph $g(x) = \begin{cases} 2x + 4 & \text{if } x < -2 \\ -3x + 2 & \text{if } x \geq -2 \end{cases}$.
- The domain of the function is split at $x = -2$. Use a table of values to graph both pieces.

x	$g(x) = 2x + 4$	$g(x) = -3x + 2$
-4	-4	
-3	-2	
-2	0	8
-1		5
0		2

Use an open circle at $(-2, 0)$ and a closed circle at $(-2, 8)$.



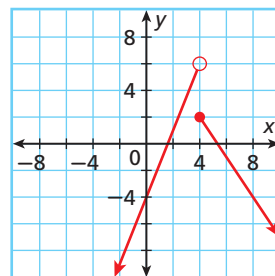
EXERCISES

7. Evaluate $f(x) = \begin{cases} \sqrt{5x + 9} & \text{if } x \geq 4 \\ 9 - 7x & \text{if } x < 4 \end{cases}$ for $x = -6$ and $x = 8$.

Graph each function.

8. $f(x) = \begin{cases} 2x - 4 & \text{if } x < 0 \\ 5 & \text{if } x \geq 0 \end{cases}$
9. $g(x) = \begin{cases} \frac{3}{2}x - 1 & \text{if } x \leq 2 \\ \sqrt{x + 2} & \text{if } x > 2 \end{cases}$

10. Write a piecewise function for this graph.



11. A bicycle delivery service charges \$6 to deliver a package that weighs 8 ounces or less. For each additional ounce, the service charges \$1.50 per ounce. Write a piecewise function for the amounts that this company charges to deliver packages that weigh 3 pounds or less.

9-3 Transforming Functions (pp. 672–679)

EXAMPLE

- Given $f(x) = \begin{cases} 2x - 2 & \text{if } x \leq 3 \\ -4x + 16 & \text{if } x > 3 \end{cases}$, write the rule for $g(x)$, a horizontal translation of $f(x)$ 5 units left.

Each piece of $f(x)$ must be shifted 5 units left. Replace every x with $(x + 5)$, and simplify.

$$\begin{aligned} g(x) = f(x + 5) &= \begin{cases} 2(x + 5) - 2 & \text{if } (x + 5) \leq 3 \\ -4(x + 5) + 16 & \text{if } (x + 5) > 3 \end{cases} \\ &= \begin{cases} 2x + 8 & \text{if } x \leq -2 \\ -4x - 4 & \text{if } x > -2 \end{cases} \end{aligned}$$

EXERCISES

12. Given $f(x) = \begin{cases} 2x - 2 & \text{if } x \leq 3 \\ -4x + 16 & \text{if } x > 3 \end{cases}$, write the rule for $h(x)$, a vertical translation of $f(x)$ 2 units up.
13. Given $f(x) = \begin{cases} 3x + 2 & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases}$, write the rule for $g(x)$, a horizontal translation of $f(x)$ 7 units right.
14. Given $f(x) = 2x^2 + 1$ and $g(x) = f\left(\frac{1}{2}x\right) + 1$, graph $g(x)$.

9-4 Operations with Functions (pp. 682–688)

EXAMPLES

Given $f(x) = x + 3$ and $g(x) = x^2 - 9$, find each function.

■ $\left(\frac{g}{f}\right)(x)$

$$\begin{aligned}\left(\frac{g}{f}\right)(x) &= \frac{g(x)}{f(x)} = \frac{x^2 - 9}{x + 3} \\ &= \frac{(x + 3)(x - 3)}{x + 3} = x - 3, x \neq -3\end{aligned}$$

■ Given $f(x) = x + 6$ and $g(x) = \frac{18}{x + 4}$, find $g(f(x))$. State its domain.

$$\begin{aligned}g(f(x)) &= g(x + 6) && \text{Substitute the rule for } f \text{ into } g. \\ &= \frac{18}{(x + 6) + 4} && \text{Use the rule for } g. \\ &= \frac{18}{x + 10}\end{aligned}$$

The domain of $g(f(x))$ is $\{x | x \neq -10\}$ because the function is undefined at $x = -10$.

EXERCISES

Given $f(x) = x^2 - 5x - 14$ and $g(x) = x - 7$, find each function.

15. $(f + g)(x)$ 16. $(f - g)(x)$

17. $(g - f)(x)$ 18. $(fg)(x)$

19. $\left(\frac{f}{g}\right)(x)$ 20. $\left(\frac{g}{f}\right)(x)$

Let $f(x) = x - 2$ and $g(x) = \frac{8}{x + 1}$.

21. Find $f(g(-2))$ and $g(f(-2))$.

22. Find $f(g(1))$, and $g(f(1))$.

23. Find $g(f(x))$, and state its domain.

24. Find $f(g(x))$ and state its domain.

25. Because of high fuel costs, an airline begins adding a fuel surcharge of \$30 to the price of each airline ticket the airline sells. Also, the airline must add 9% to the price for airport and sales taxes. Write a composite function for how much a person would pay for a ticket with this airline that is x dollars before surcharges and taxes.

9-5 Functions and Their Inverses (pp. 690–696)

EXAMPLES

■ Find the inverse of $f(x) = -3(x - 6)^2$. Determine whether it is a function, and state its domain and range.

$$y = -3(x - 6)^2 \quad \text{Rewrite the function by using } y.$$

$$x = -3(y - 6)^2 \quad \text{Switch } x \text{ and } y \text{ in the equation.}$$

$$-\frac{x}{3} = (y - 6)^2 \quad \text{Divide both sides by } -3.$$

$$\pm\sqrt{-\frac{x}{3}} = y - 6 \quad \text{Take the square root of both sides.}$$

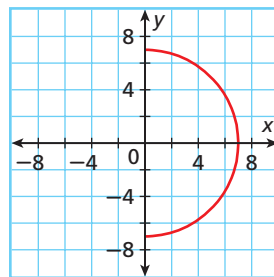
$$y = \pm\sqrt{-\frac{x}{3}} + 6 \quad \text{Simplify.}$$

$$f^{-1}(x) = \pm\sqrt{-\frac{x}{3}} + 6 \quad \text{Rewrite as } f^{-1}(x).$$

Because there is a positive y -value and a negative y -value for any $x < 0$, the inverse is not a function. Because the radicand must be greater than or equal to 0, the domain is $\{x | x \leq 0\}$. The range is $\mathbb{R}$.

EXERCISES

26. Use the horizontal-line test to determine whether the inverse of the relation graphed is a function.



Find the inverse of each function. Determine whether the inverse is a function, and state its domain and range.

27. $f(x) = 5 - 8x$

28. $f(x) = \left(\frac{1}{3}x + 2\right)^2$

29. $f(x) = \frac{5}{2x + 8}$

30. $f(x) = 3 + \sqrt{x - 5}$

EXAMPLES

- Determine by composition whether $f(x) = \frac{1}{3}x - 4$ and $g(x) = 12 + 3x$ are inverses.

Find both compositions.

$$f(g(x)) = \frac{1}{3}(12 + 3x) - 4 = 4 + x - 4 = x$$

$$g(f(x)) = 12 + 3\left(\frac{1}{3}x - 4\right) = 12 + x - 12 = x$$

Because $f(g(x)) = g(f(x)) = x$, f and g are inverses.

EXERCISES

Determine by composition whether each pair of functions are inverses.

31. $f(x) = 3x - 5$ and $g(x) = \frac{x-3}{5}$

32. $f(x) = \sqrt[3]{x-5}$ and $g(x) = x^3 + 5$

33. The formula for the surface area of a sphere with radius r is $A(r) = 4\pi r^2$. Find and interpret the inverse of $A(r)$.

9-6 Modeling Real-World Data (pp. 698–705)

EXAMPLE

- The table shows the ticket prices to a minor league baseball game in relation to the number of years since the team began playing.

Baseball Ticket Prices	
Year	Price (\$)
1	9.50
2	10.25
3	11.10
4	12.00
5	12.92

Step 1 Check the **first differences** of the prices.

$$0.75 \quad 0.85 \quad 0.90 \quad 0.92$$

Because the first differences are not constant, a linear model is not a good fit.

Step 2 Check the **second differences**.

$$0.10 \quad 0.05 \quad 0.02$$

Because the second differences are not constant, a quadratic model is not a good fit.

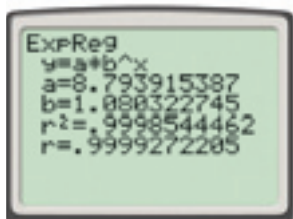
Step 3 Check the **ratios**.

$$\frac{10.25}{9.5} \approx 1.08, \frac{11.10}{10.25} \approx 1.08, \frac{12}{11.10} \approx 1.08, \frac{12.92}{12} \approx 1.08$$

The ratios are close to 1.08. An exponential model is a good fit.

Step 4 Perform an exponential regression.

An appropriate model is $f(x) = 8.79(1.08)^x$.



EXERCISES

34. The table shows the city of Culver's water use in relation to daily high temperature.

Water Use in Culver	
Daily High Temperature (°F)	Water Use (million gal)
55	71.3
60	78.7
65	86.9
70	96
75	106
80	117

- Find an appropriate model for this data. Use temperature t as the independent variable.
- Use your model to predict the number of gallons that Culver will use when the high temperature is 85°F .
- Use your model to predict the high temperature when the water use is 50 million gallons.

CHAPTER TEST

- James receives a salary of \$300 per week plus a commission of 3% of the amount he sells. Create a table, a graph, and an equation to represent his weekly earnings on sales of 0 to 10,000 dollars.
- While standing at the top of a cliff, Kurt accidentally knocks a stone loose. The table shows the height of the stone in meters after t seconds.
 - Create a graph and an equation for the data by using time t as the independent variable.
 - How high is the cliff?
 - Find the height of the stone after 10 seconds.
 - When will the stone hit the ground?

Height of Falling Stone	
Time (s)	Height (m)
1	615.1
2	600.4
3	575.9
4	541.6
5	497.5
6	443.6

Graph each function.

$$3. f(x) = \begin{cases} -x - 3 & \text{if } x < 1 \\ 2x - 6 & \text{if } x \geq 1 \end{cases}$$

$$4. g(x) = \begin{cases} 5 & \text{if } x \leq -2 \\ -x^2 - 4x & \text{if } x > -2 \end{cases}$$

Given $f(x)$, graph $g(x)$.

$$5. f(x) = 2x - 4 \text{ and } g(x) = -\frac{1}{2}f(x) - 1$$

$$6. f(x) = x^2 - 2 \text{ and } g(x) = -f(x + 2)$$

Given $f(x) = 4x^2 - 9$ and $g(x) = 2x + 3$, find each function or value.

$$7. (f - g)(4)$$

$$8. g(f(3))$$

$$9. (fg)(5)$$

$$10. \left(\frac{g}{f}\right)(x)$$

- Ramon pays a 10% insurance fee for each piece of jewelry in his store. He then prices the item for sale at 150% of his total cost. Write a composite function for the price of an item with an original cost of c dollars.

Write the rule for the inverse of each function. Determine whether the inverse is a function, and state its domain and range.

$$12. f(x) = 12 - 5x$$

$$13. g(x) = \frac{10}{x + 4}$$

$$14. h(x) = \frac{(x + 5)^2}{2}$$

- The table shows the average sales prices of houses and the houses' distances from downtown.
 - Find an appropriate model for the data by using distance d as the independent variable.
 - Use your model to predict the average sales prices of houses that are 20 miles from downtown.

Sales Prices of Houses	
Distance from Downtown (mi)	Average Sales Price (\$)
2	118,496
4	109,016
6	100,295
8	92,271
10	84,890
12	78,098

COLLEGE ENTRANCE EXAM PRACTICE

FOCUS ON SAT STUDENT-PRODUCED RESPONSES

Some questions on the SAT require you to enter your answer in a special grid. Your answers must be positive integers, fractions, or decimals. You cannot enter negative numbers or mixed numbers in the grid.



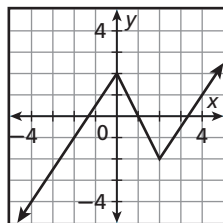
Some questions may have multiple answers; in these cases you may enter any one correct answer. If the solution is an inequality, be sure that you choose a number from the solution region.

.	/	/	/
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

You may want to time yourself as you take this practice test. It should take you about 9 minutes to complete.

1. If 5 less than 3 times a number is equal to 2 more than twice the number, what is the number?

2. The graph of $f(x)$ is shown.



If $g(x) = -f(x) + 1$, what is $g(2)$?

3. Give a possible value for x in the inequality $-4(2x - 3) > 4x - 24$.

4. Let the operations $\blacklozenge$ and $\heartsuit$ be defined for real numbers a and b as shown.

$$a \blacklozenge b = 2a - b$$

$$a \heartsuit b = \frac{a + b}{2}$$

What is the value of $(4 \heartsuit 9) \blacklozenge 3$?

5. Maria drove to her grandmother's house at an average speed of 60 miles per hour. On the way home, she averaged only 45 miles per hour due to traffic. If she spent a total of $3\frac{1}{2}$ hours driving, how many miles is the trip to her grandmother's house?

6. The table shows some values for the function f .

x	-2	0	2	4
$f(x)$	7	4	1	-2

What is the value of $f^{-1}(-2)$?



Multiple Choice: Eliminate Answer Choices

With some multiple choice test items, you can use mental math or logic to quickly eliminate some of the answer choices before you begin solving the problem.

EXAMPLE 1

1

Tyler can install an air conditioning unit in 3 hours. If Laura helps him, the job is done in 2 hours. How many hours would it take Laura working alone?

- (A) 1 hour (C) 6 hours
(B) 2 hours (D) 8 hours

READ the question. Then try to eliminate some of the answer choices.

Use logic:

When Tyler works alone, the job gets done in 3 hours. When working with Laura, the job takes only 2 hours. So, it is reasonable to assume that Laura working alone takes MORE THAN 2 hours to complete the job.

Based on this logic, **eliminate** choices A and B.

Set up and solve a rational equation to find the correct answer, C.

EXAMPLE 2

2

Ryanne swims six days a week. Her coach starts keeping time when Ryanne starts warming up and notes how long Ryanne has been at the pool after every 2 laps. The table shows the time that it takes for Ryanne to swim 12 laps. If Ryanne wants to swim 24 laps, how long will it take?

- (F) 28 minutes (H) 48 minutes
(G) 38 minutes (J) 50 minutes

Laps	Time (min)
2	6
4	10
6	14
8	18
10	22
12	26

LOOK at the data, and eliminate some answer choices.

Use mental math and logic:

From the data in the table, you can tell that Ryanne swims 2 laps every 4 minutes. So it takes her 2 minutes to swim 1 lap.

So 24 laps would take $24(2) = 48$ minutes. You can eliminate any answer choice that is LESS THAN 48 minutes: choices F and G.

Before you select choice H as your answer, be careful. Look at the data in the table again. The first 2 laps that Ryanne swims take her 6 minutes, not 4 minutes, so your estimate of 48 laps is a bit low. Therefore, **eliminate** choice H. Choice J is the correct answer.



Try to eliminate unreasonable answer choices. Some choices may be too great or too small or may have incorrect units.

Read each test item and answer the questions that follow.

Item A

The width of a rectangle is 6 feet less than its length. Which of the following systems of equations can be used to find the dimensions of the rectangle if the perimeter of the rectangle is 56 feet?

- (A) $\ell = w - 6$
 $2\ell + 2w = 56$
- (B) $w = \ell - 6$
 $2(\ell + w) = 56$
- (C) $\ell = w - 6$
 $\ell w = 56$
- (D) $w = \ell - 6$
 $\ell w = \ell 6$

- What is the perimeter formula for the area of a rectangle? Based on this formula, are there any choices that you can eliminate immediately? If so, which choices and why?
- Read the first sentence of the test item again and write an expression. Are there any more answer choices that you can eliminate? Explain.

Item B

The volume V of a gas varies inversely with the pressure P and directly with the temperature T . A certain gas has a volume of 30 liters, a temperature of 345 kelvins, and a pressure of 1 atmosphere. If the gas is compressed to a volume of 20 liters and heated to 375 kelvins, what will the new pressure be?

- (F) 0.72 atmosphere (H) 1.5 atmospheres
- (G) 0.72 liter (J) 1.63 atmospheres

- Are there any answer choices that logically do not make sense and can be eliminated? If so, which choices and why.
- Because the volume of a gas varies inversely with the pressure, if the volume decreases, should the pressure increase or decrease? Can you eliminate any of the answer choices by using this information?

Item C

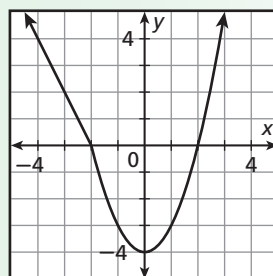
A moving truck company charges \$125 a day for driving its truck 50 miles or less. The company charges an additional \$0.05 per mile for all miles driven over 50 miles. Which of the following functions represents the fee for this moving truck for x miles in a day?

- (A) $C(x) = \begin{cases} 125 & \text{if } 0 \leq x \leq 50 \\ 2.5 & \text{if } x > 50 \end{cases}$
- (B) $C(x) = \begin{cases} 125 & \text{if } 0 \leq x \leq 50 \\ 125 + 0.05(x - 50) & \text{if } x > 50 \end{cases}$
- (C) $C(x) = \begin{cases} 50 & \text{if } 0 \leq x \leq 50 \\ 50 + 0.05x & \text{if } x > 50 \end{cases}$
- (D) $C(x) = \begin{cases} 0.05 & \text{if } 0 \leq x \leq 50 \\ 125x & \text{if } x > 50 \end{cases}$

- Look at answer choice A. Why can it be eliminated immediately?
- Sarah wants to eliminate choice C. Do you agree? Explain.

Item D

Which function corresponds to the graph?



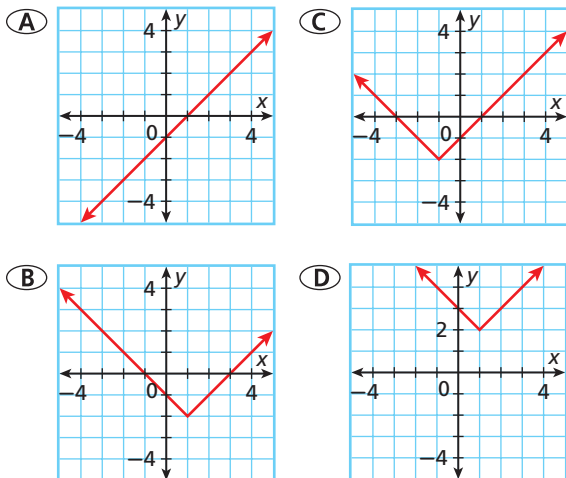
- (F) $g(x) = \begin{cases} x^2 - 4 & \text{if } x \geq 0 \\ -2x - 4 & \text{if } x < 0 \end{cases}$
- (G) $g(x) = \begin{cases} x - 4 & \text{if } x \geq -2 \\ -2x & \text{if } x < -2 \end{cases}$
- (H) $g(x) = \begin{cases} x^2 - 4 & \text{if } x \geq -2 \\ -2x - 4 & \text{if } x < -2 \end{cases}$
- (J) $g(x) = \begin{cases} x^2 & \text{if } x \geq -2 \\ -2x + 4 & \text{if } x < -2 \end{cases}$

- Describe the functions on the graph. Which answer choice can be eliminated based on the shape of the function?
- Kaye looked at the domain of the function and decided to eliminate choice F. Do you agree with Kaye's decision? Explain.

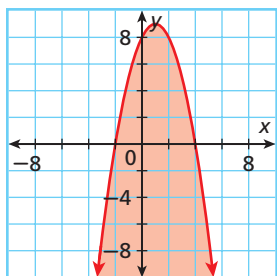
CUMULATIVE ASSESSMENT, CHAPTERS 1–9

Multiple Choice

1. Which is the graph of $f(x) = |x + 1| - 2$?



2. Which equation or inequality best represents the graph?



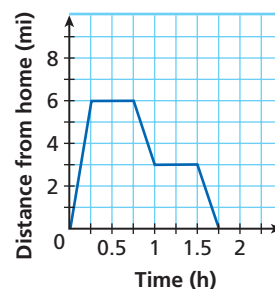
- (F) $y = x^2 + 2x + 8$
 (G) $y = -x^2 + 2x + 8$
 (H) $y \leq x^2 + 2x + 8$
 (J) $y \leq -x^2 + 2x + 8$
3. Which is the augmented matrix for this system of equations?

$$\begin{cases} -5y = 8 - x \\ y + 3x = 10 \end{cases}$$

- (A) $\left[\begin{array}{cc|c} -5 & 8 & -1 \\ 1 & 3 & 10 \end{array} \right]$ (C) $\left[\begin{array}{cc|c} 1 & -5 & 8 \\ 1 & 3 & 10 \end{array} \right]$
 (B) $\left[\begin{array}{cc|c} 1 & -5 & 8 \\ 3 & 1 & 10 \end{array} \right]$ (D) $\left[\begin{array}{cc|c} -5 & 8 & -1 \\ 3 & 1 & 10 \end{array} \right]$

4. Which description best reflects the graph shown?

Lou's Distance from Home



- (F) Lou drove 6 miles to the library, spent an hour there, and then drove straight home.
 (G) Lou drove 6 miles to the library, spent half an hour there, stopped by the video store for half an hour, and then drove home.
 (H) Lou drove 3 miles to the library, spent half an hour there, drove another 3 miles to the movie store, and then drove home.
 (J) Lou drove 6 miles to the library, spent half an hour there, drove another 3 miles to the video store, and spent an hour there.

5. Evaluate the piecewise function for $x = -1$.

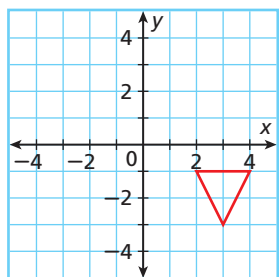
$$f(x) = \begin{cases} x^2 + 4x - 8 & x < -1 \\ x^3 - x^2 + 5 & x \geq -1 \end{cases}$$

- (A) -13 (C) 3
 (B) -11 (D) 5
6. Solve for x .
 $\sqrt{2x - 4} = x - 6$
 (F) $x = 10$ (H) $x = 2$ and $x = 20$
 (G) $x = 4$ and $x = 10$ (J) $x = 2$ and $x = 12$

7. Given $f(x) = 2x^2 - 7x - 30$ and $g(x) = x - 6$, find $\left(\frac{f}{g}\right)(x)$.

- (A) $2x - 5$ (C) $\frac{(2x - 5)(x + 6)}{x - 6}$
 (B) $2x + 5$ (D) $\frac{(2x - 10)(9x + 3)}{x - 6}$

8. Which transformation of triangle ABD creates an image with a vertex at $(-2, 1)$?



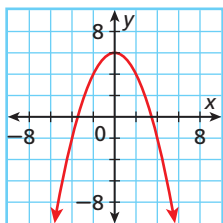
- (F) Reflect $\triangle ABC$ across the x -axis.
 (G) Reflect $\triangle ABC$ across the y -axis.
 (H) Translate $\triangle ABC$ 3 units left and 3 units up.
 (J) Rotate $\triangle ABC$ 180° about the origin.



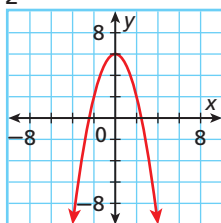
In Item 9, examine one part of the function at a time, eliminating answer choices until you find a graph that matches all parts of the function.

9. Which is the graph of $f(x) = -\frac{1}{2}x^2 + 6$?

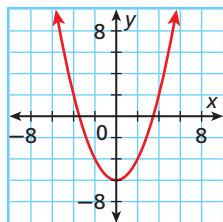
(A)



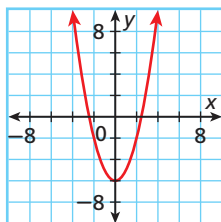
(C)



(B)



(D)



Gridded Response

10. Find the value of $\log_4 256^3$.
11. What is the value of c in the given equation?
 $4(5i - 2) + 3 = 2(10i + c) - 7$
12. Find the value of the given expression when $x = 5$.

$$\left(\frac{x^2 + 5x - 36}{2x^2 - 10x + 8} \right) \left(\frac{x^2 + x - 2}{x^2 + 11x + 18} \right)$$

Short Response

13. The equation $f(x) = x^2 + 1$ is a function.
- Find the inverse of the function.
 - Graph $f(x) = x^2 + 1$ and its inverse.
 - Explain whether the inverse is a function.
14. Use the points below.
 $(0, 6)$, $(2, 2)$, and $(5, 11)$
- Write a quadratic function that fits the points.
 - Check the quadratic function that you wrote by substituting the ordered pairs. Verify that each is a solution.
 - Graph the equation.
 - Find $f(7)$ and $f(-7)$.
15. Consider the function $f(x) = x^2 - 4$.
- Identify two different transformations of f so that the vertex would be $(1, 4)$.
 - Identify two different transformations of f so that its graph would pass through $(0, 2)$ and $(-4, 2)$.

Extended Response

16. The volume of gas in a car depends on the number of miles that have been driven since the tank was last filled.

Distance driven (mi)	0	50	100	150	200
Gas (gal)	10	8	6	4	2

- Use constant differences or ratios to determine which parent function would best model the given data.
- Write the equation for the data.
- How many gallons are left after 75 miles?
- Can the car be driven for 300 miles? Why or why not?
- Find and interpret the inverse of the equation.